OpenMath Content Dictionaries

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April 15, 2002

Bug fixes: differentiation

`<OMS name="diff" cd="calculus1"/>` denotes a single differentiation. However, we cannot express $\frac{\mathrm{d}^n}{\mathrm{d}x^n}$. The network is therefore proposing to the OpenMath Society that the `calculus1` CD be extended by an `nthdiff` symbol, as follows for $\frac{\mathrm{d}^2}{\mathrm{d}x^2}x^3$.

```
<OMA>
  <OMS name="nthdiff" cd="calculus1"/>
  <OMI> 2 </OMI>
  <OMBIND>
    <OMBVAR>
      <OMV name="x"/>
    </OMBVAR>
    <OMA>
      <OMS name="power" cd="arith1"/>
      <OMV name="x"/>
      <OMI> 3 </OMI>
    </OMBIND>
</OMA>
```

General extensions proposed.

- A `transc2` CD, with a binary `arctan` operator corresponding to the FORTRAN `atan2` function. Added functionality.

- Extensions to `list2`: `reverse`, `append`, `nth` — possibly more..

- `logic3` to encode formal proofs and deductions. More probably needed here.

Changes to Polynomials

`poly` Add `leading_coefficient` and `coefficient`.

`polyd` Add a new constructor `poly_ring_d_named` which names (rather than numbers) the variables.

Add new attribute `completely_reduced`

Rename `groebner_basis` as `groebnered`.

Add new orderings `matrix_ordering` and `weighted`.

Add a new operation `weighted_degree`.

Units and Dimensions

- `dimensions1` CD — essentially complete.

- `units_metric1` CD — largely complete but more FMPs need adding.

- `units_imperial1` CD — many FMPs supporting conversion into metric need adding.

Special Functions — Infrastructure

`cauchypv` **CD** Symbols `defint` and (locating the problems) `defint_singularities`.

`odesoln1` **CD** Symbol `ODEsolution`:

1. two bound variables: the solution and the independent variable;

2. The ordinary differential equation;

3. List of initial conditions.

`contour1` **CD** Symbol `path_description`, as (constraints on) a function $[0, 1] \to \mathbf{C}$.

How to encode analytic continuation?

Special functions — examples

`airy` **CD** `Ai, Ai2, Bi, Bi2.`

`errorint` **CD** `erf, erfc, FresnelS, FresnelC, w.`

`gamma` **CD** `Gamma` (so far). Problems found in Abramowitz & Stegun.

`Bessel` **CD** `J` and `Y`. How many other variants should we encode?

Future work on CDs: Abstract Algebra

Many more CDs need to be written and/or formalised in this area. Here too, there are problems of consistency:

Degree S_{12}, M_{12} are permutation groups acting on 12 symbols;

Size F_{20} is a permutation group of size 20, normally acting on 5 elements;

Unclear Is D_{12} a group with 12 elements acting on six points, or a group with 24 elements acting on 12 points? One can find both in the literature, though the second is probably more common. It would make sense to follow Conway *et al.*

OpenMath also needs to deal with ideals etc., rather than just lists of polynomials (different lists can represent the same ideal).

Future Work on CDs: other

Algorithms A CD to describe algorithmic concepts would be useful, partly from the point of view of the wider publication-related aspects of OpenMath, and partly for use in concepts such as symbolic differentiation, i.e. differentiating an algorithm, where co-operation between software packages is important.

Logics While basic classical logic (propositional, predicate) is catered for, there is nothing on more general proofs, or other forms of logic (intuitionistic etc.). Different concepts of equality also need to be handled.

Special Functions Much more work needs to be done here, but this should become easier once the infrastructural issues are solved.