Digital Library of Mathematical Functions: LaTeX, MathML and . . . OpenMath?

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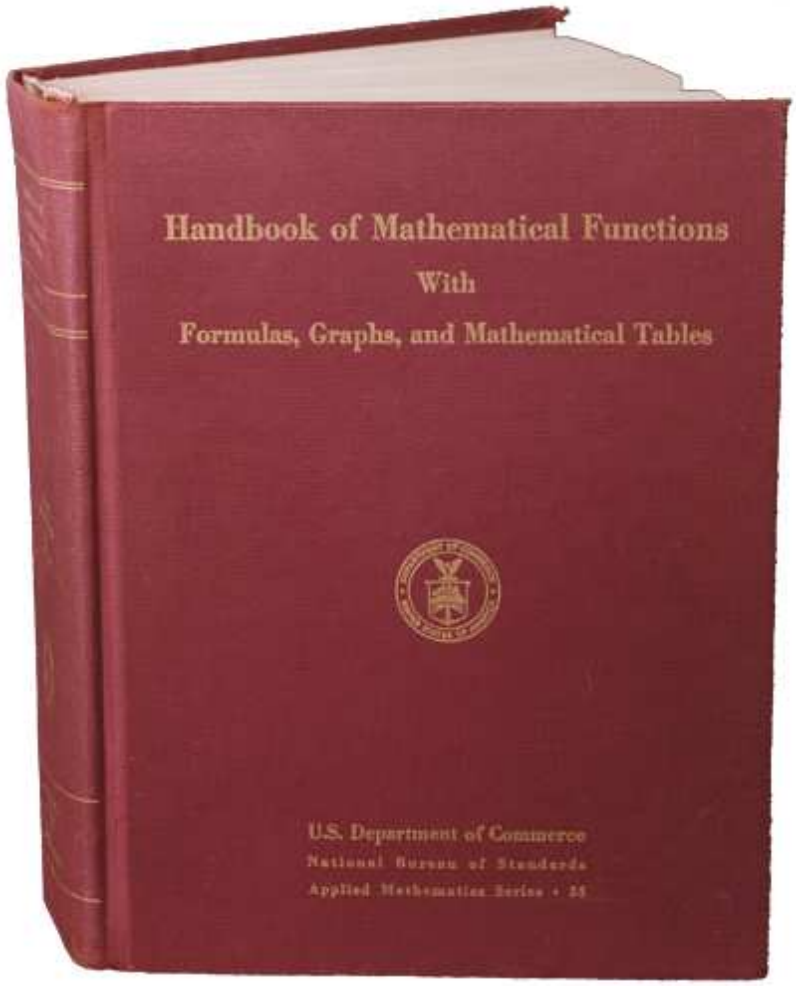
Digital Library of Mathematical Functions

Needing no introduction...



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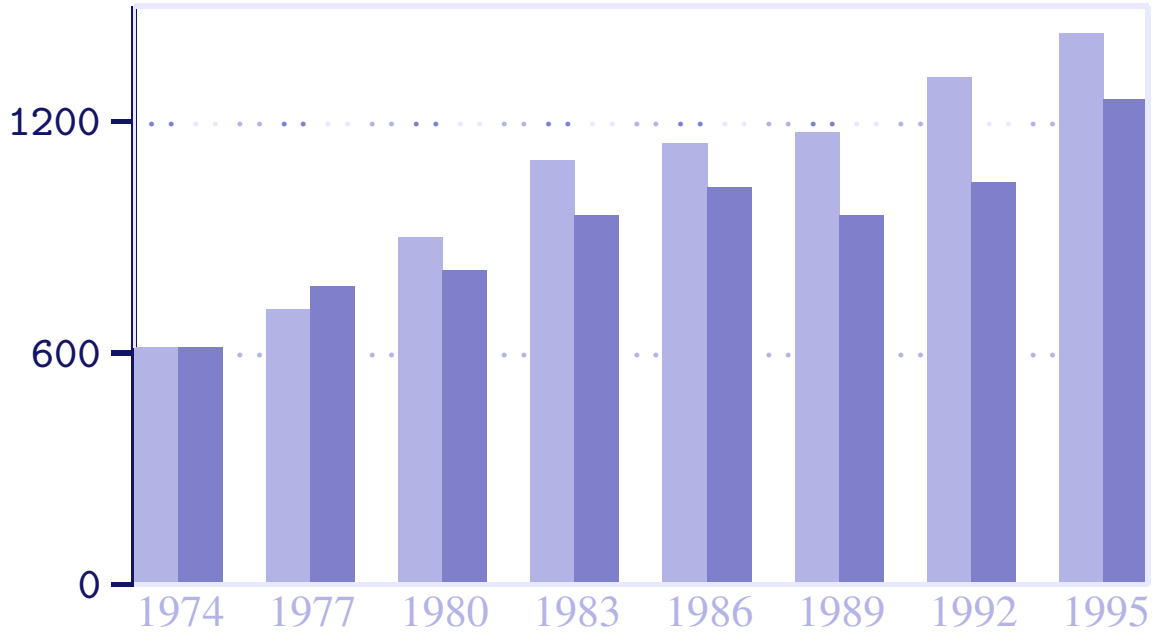
Old, but still relevant

Citations of AMS55 relative to All Scientific.



AMS55 is apparently used more than ever.

Time for a Rewrite

- New functions;
- New properties of old functions;
- New applications.
- . . . and many opportunities.
  - The Internet;
  - Computer Algebra, Theorem Proving systems;
  - The Semantic Web.

DLMF Project

- Started looking at feasibility in 1997.
- NSF funding for authorship in 1999.
- 4 editors, $\approx$ 12 associate editors, $\approx$ 40 authors.
- Goals:
  - New mathematical content updating AMS55,
  - in form of Digital Library,
  - and in print form,
  - by 2005.

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Choices: LaTeX, XML, MathML, OpenMath

- LaTeX is obviously good choice for document source.
- . . . and obviously bad.
- Target: XML, MathML, and (eventually) OpenMath.

I don't need to tell *you* why. . .

Overview of talk

- LaTeXML tool.
- Metadata: markup, annotations and connections,
- Data model of the Library
- Math: Parsing, synthesizing meaning.

LaTeXML: Goals

- LaTeX ⇒ XML Transformer
  - General purpose.
  - LaTeX-like DTD (or other?)
  - Math to MathML, OpenMath
- Closely mimic TeX behaviour (& Quirks).
- Lossless.
- Extensible, Adaptable.
- Encourage higher-level markup, declarations.
- . . . and finish DLMF project!

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LaTeXML: DLMF Approach

To make more feasible adopt

- Modestly Content-oriented LaTeX.
- Discourage Presentation Markup
  but don't forbid.
- Encourage Content Markup,
  but keep typeable.
- Use document-specific information (internal/external)
  to resolve ambiguities.

Metadata: Making Connections

- Traditional LaTeX: `\ref`, `\cite`, `\index`.
- Leverage our mathematics markup.
- Additional markup:
  - Annotations `\note`.
  - Special metadata: Original handbook reference.
  - Additional declarations.

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Metadata: Using Connections

- Postprocessing XML documents.
- Disassemble XML into ‘database’.
- Note all connections.

Not really that hard.

DLMF Data Model

- Simple model (maybe too simple)
  - ID $\Rightarrow$ Object(XML)
    (Chapter, Section, Table, Equation, . . .)
  - linkages embedded within each object
    (insertion, reference, . . .)
- Can (re)construct as necessary
  - Sectional units,
  - Search 'hit-lists'
- Developing an 'Indexing' API by which search, refnum lookup, . . .
  $\Rightarrow$ ID's

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LaTeXML Math Processing

TeX source $\xrightarrow{\text{LaTeXML}}$ XML

- Let LaTeXML deal with TeX quirks.
- Acts as structure-preserving Lexer.
  - Possibly augmented (math) Tokens:
    - Name,
    - Unicode, Font, ...
    - PartOfSpeech (ID, Function, Operator, ...)
    - Type (eventually).
  - preserve any given structure (eg. `\frac`, ...)

Math: The Easy Stuff

```
a = b+c
```

LaTeXML produces the tokens

```
<XMTok>a</XMTok>
<XMTok>=</XMTok>
<XMTok>b</XMTok>
<XMTok>+</XMTok>
<XMTok>c</XMTok>
```

LaTeXML Math Processing *continued*

XML $\xrightarrow{\text{LaTeXMLpost}}$ XML'

- Grammar-based parser.
- Undeclared tokens get PartOfSpeech from
  - Document-specific dictionary
    (possibly sectionally scoped)
  - Default dictionary
- Resulting Expression tree
  - inspired by OpenMath.
  - $\approx$ Content MathML;
    (although we haven't done this yet).
  - Easily converted to Presentation MathML.

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Math: The Easy Stuff *continued*

```
a = b+c
```

LaTeXMLpost parses this into

```
<XMApp><XMTok>=</XMTok>
  <XMTok>a</XMTok>
  <XMApp><XMTok>+</XMTok>
    <XMTok>b</XMTok>
    <XMTok>c</XMTok>
  </XMApp>
</XMApp>
```

Math: The Easy Stuff *continued*

```
a = b+c
```

Conversion to MathML yields

```
<math xmlns="http://www.w3.org/1998/Math/MathML">
  <mrow>
    <mi>a</mi>
    <mo>=</mo>
    <mrow>
      <mi>b</mi>
      <mo>+</mo>
      <mi>c</mi>
    </mrow>
  </mrow>
</math>
```

LaTeXML Math Processing *future*

XML' $\xrightarrow{\text{LaTeXMLpost}}$ XML"

- Extension of Dictionary to support some Type system.
- Type Analysis to further resolve 'meaning'
- $\Longrightarrow$ OpenMath.
- Any advice?

Math: Higher Level Markup

Reduce ambiguities by introducing higher-level markup:

$$\texttt{\textbackslash deriv[n]\{f\}\{x\}} \Rightarrow \frac{d^n f(x+y)}{dx^n}$$

LaTeX code:

omitted

LaTeXML declaration:

```
DefConstructor('\deriv[]{}{}',
       "<XMApp !#2(POS='BIGOP')>"
       . "<XMTok name='deriv'/>"
              . "?#2(<XMArg>#2</XMArg>)!#2(<XMTok name='Empty'
       . "<XMArg>#3</XMArg>"
              . "?#1(<XMArg>#1</XMArg>)</XMApp>");
```

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Math: Higher Level Markup *continued*

LaTeXML constructs the tree:

```
<XMApp><XMTok name='deriv'/>
  <XMArg><XMTok>f</XMTok>
         <XMTok>(</XMTok>
         <XMTok>x</XMTok>
         <XMTok>+</XMTok>
         <XMTok>y</XMTok>
         <XMTok>)</XMTok>
  </XMArg>
  <XMArg><XMTok>x</XMTok></XMArg>
  <XMArg><XMTok>n</XMTok></XMArg>
</XMApp>
```

Parser can treat args individually,

. . . avoiding much guesswork.

Math: Special Functions

With appropriate TEX macrology:

$$\texttt{\textbackslash HyperpFq\{p\}\{q\}} \Rightarrow {}_pF_q$$

Introduce notion of *evaluating a function at*:

$$\texttt{\textbackslash HyperpFq\{p\}\{q\}@\{a\}\{b\}\{z\}} \Rightarrow {}_pF_q\left(a;b;z\right)$$

or (alternative notation)

$$\texttt{\textbackslash HyperpFq\{p\}\{q\}@@\{a\}\{b\}\{z\}} \Rightarrow {}_pF_q\left(\begin{matrix}a\\b\end{matrix};z\right)$$

Palatable notation? Easier to type than

```
\sideset{_{p}}{_{q}}{\mathop{F}}\left({a \atop b};z\right)
```

Math: Special Functions *continued*

With the end result:

```
<XMApp>
  <XMTok name='HyperpFq'>F</XMTok>
  <XMTok>p</XMTok>
  <XMTok>q</XMTok>
  <XMTok>a</XMTok>
  <XMTok>b</XMTok>
  <XMTok>z</XMTok>
</XMApp>
```

and we know *which* 'F' is intended.

Math: Issues

- Role of text and spacing in math.
- Overloading of *symbols* (scoping?)
  - f is a function here, but a variable there.
- Palatable content math markup for LaTeX.
- For *really* meaningful math (eg. OpenMath)
  - need type analysis
  - need more info from authors
- Open ended...

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Trends? (Or Wishes)

- Continued development and support for MathML
- Ditto for OpenMath
- Convergience of Markup styles and DocTypes for
  - Various LaTeX⇒ XML converters
  - Richer LaTeX content markup in general (LaTeX3?)
  - Project Authors able use different tools LaTeX, CAS, Thm.Provers, Word Processors.

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