A language for brokering mathematical services

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We propose a software framework for brokering mathematical services that are distributed among networked servers. The foundation of this framework is a language for describing the mathematical problems solved by the services. Servers register their problem solving capabilities with a "semantic broker" to which clients submit corresponding task descriptions. The broker (possibly in cooperation with a deduction system) determines the suitable services and returns them to the client for invocation. This mechanism thus hides from the client the actual implementation of mathematical services.

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  input must satisfy additional service requirements

  output must satisfy additional client requirements

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- Definitions
- Theorems
- Proofs

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Semantic Web of Mathematics is intended as a collection of mathematical services operating with/on formal resources

Mathematics on the Web is mainly geared to human consumption

There is almost no mathematical resource or service that is *meant to* or *can be* processed automatically:

- ◇ machine-readable (representation standards)
- ◇ machine-understandable (meta-information on properties)

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GOAL

make resources and *services* in particular usable as black bloxes
no human insight needed

xml, Estensible Markup Language, is a structured format for documents and data.
Applications:

- MathML, Mathematical Markup Language
- RDF, Resource Description Framework
- WSDL, Web Services Description Language
- CDL, Conversation Definition Language
- WSFL, Web Services Flow Language
- ...

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user/client must know the service name in order to use it

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(-) Abstraction from service functionality:
user/client need insight on protocol, complexity, availability . . .

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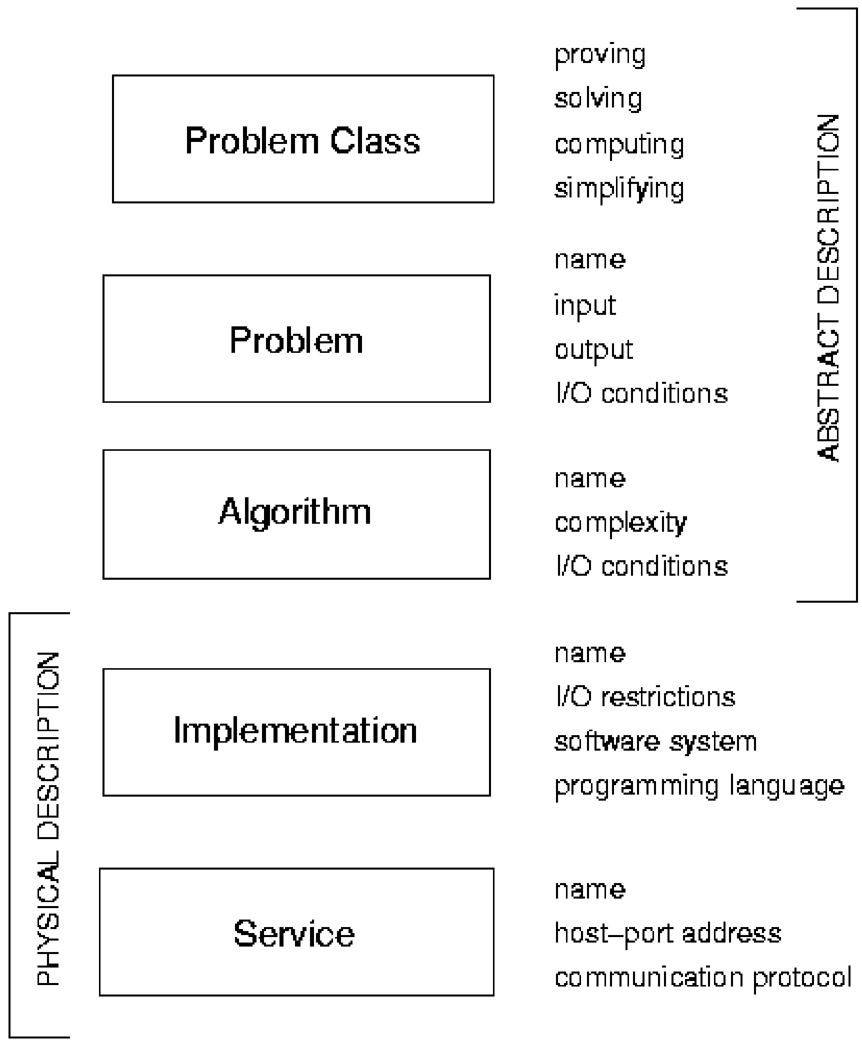
  service endpoints, communication protocols
- Semantic Behavior
  I/O conditions, complexity
- Pragmatic Issues
  implementation limitations, machine constraints

Use a formal language to define the semantics (e.g. the meta-data) of the mathematical service:

- definitions
- domain axiomatization
- behavioral description by pre-conditions and post-conditions

additionally consider syntactical and pragmatic issues


Problem Class
proving
solving
computing
simplifying
Problem
name
input
output
I/O conditions
Algorithm
name
complexity
I/O conditions
ABSTRACT DESCRIPTION
Implementation
name
I/O restrictions
software system
programming language
Service
name
host–port address
communication protocol
PHYSICAL DESCRIPTION

Layers

- ◇ divide and conquer, break down problem
- ◇ do the pragmatics first, specify the theory later
- ◇ be open to the world, incorporate available description languages

A *problem class* description defines the structure of classes of problems (parametrized by a logical system L and a theory T of L) including

◇ *Computing*, carrying out operations in a domain, i.e., given a term f with a free variable x and a value a in a properly chosen domain, return the value of f with a substituted for x;

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- *Proving*, showing truth of a logical statement i.e., given a closed formula F in a particular logic, determine whether the formula is true in a particular domain or not. For some logic, this is equivalent to returning a proof.

A Computing Problem

```
<computingProblem
xsi="http://www.mathematics.net/problem/computing/algebra/realroot">
  <name>Real root isolation</name>
  <comment encoding="xhtml">Real root isolation of a polynomial in Q[x]</comment>
  <input>
    <parameter encoding="openmath"><OMOBJ><OMV name="p"/></OMOBJ></parameter>
    <domain encoding="openmath"><OMOBJ><OMA><OMS name="PolynomialRingR" cd="polyr"/>
                                <OMS cd="setname1" name="Q"/><OMV name="x"/>
                                </OMA></OMOBJ></domain>
  </input>
  <output>
    <parameter encoding="openmath"><OMOBJ><OMV name="l"/></OMOBJ></parameter>
    <domain encoding="openmath"><OMOBJ><OMA><OMS name="list" cd="list"/>
                                <OMA><OMS name="isolating_interval" cd="real_roots"/>
                                <OMS  cd="setname1" name="Q"/></OMA></OMA><OMOBJ>
    </domain>
  </output>
  <pre><domain="FOL" link="http://mbase.mathweb.org:8080/mbase">...p is square-free...</pre>
  <post><domain="FOL" encoding="openmath">...the elements of l are
        disjoint open isolating intervals for the roots of p... </post>
</computingProblem>
```

- Problem P specializes to problem P' (EGCD vs GCD)

  pre(P') $\longrightarrow$ pre(P), post(P') $\longrightarrow$ post(P)

- Algorithm for P' also solves P

- Algorithm for P' also solves P under additional contraints

Want P': try P

Want P: try P' with additional checks

Want P': may even try P'' with additional checks

The goal of this project is not to invent new conceptual models for the relationship between mathematical/logical theories but to provide a concrete software framework for the mathematical community. It shall be as far as possible based on (i.e., integrate and generalize) existing activities of other groups that have resulted in concrete models and/or software components (*OpenMath*, IAMC, JavaMath, MathWeb, MathBus) and make utmost use of widely accepted computing and communication standards (in particular, the XML world).

- evaluate available e-business technologies for web services (SOAP, WSDL, UDDI)
- select two or three case studies to guide the development
- XML, Java, server-side programming

Stable server: http://poseidon.risc.uni-linz.ac.at:8080/mathbroker

Development server: http://perseus.risc.uni-linz.ac.at:8080/mathbroker

Test web services: http://perseus.risc.uni-linz.ac.at:8080/openmath `http://localhost:8080/axis/services/OMObjProcessor?wsdl`

`http://localhost:8080/axis/services/SymbolicEvaluator?wsdl`

◇ AXIS http://xml.apache.org/axis/

deployed services in Axis can be accessed with a standard web browser and appending "?WSDL" to the end of the URL, generates a WSDL description of the service.
"WSDL2Java" tool builds Java proxies and skeletons for services with WSDL descriptions.
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WSDL4J and WSIL4J APIs provide an object model for WSDL documents.

UDDI4J API for publishing and finding web services in a UDDI Registry.

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It seems that WSDL is intended to model services as intended by e-business, hence a service is a collection of `ports` where each port is a specific aspect of a service transaction.

Future work has to establish how to use WSDL for modelling mathematical services in either way

- ◇ model mathematical services within WSDL by exploiting the extensionality bindings
  - ◇ (XSD, Relax?) schema for message syntactic types (OMOBJ vs OMOBJ-polys)
  - ◇ XSD schema for message as a triple: parameter, parameter type, conditions on the parameter
- ◇ design a new description language for mathematical services (MSDL) from which to generate WSDL