The OpenMath Standard
The OpenMath Esprit Consortium

## Editors

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#### Abstract

This document proposes OpenMath as a standard for the communication of semantically rich mathematical objects. This draft of the OpenMath standard comprises the following: a description of OpenMath objects, the grammar of XML and of the binary encoding of objects, a description of Content Dictionaries and an xml document type definition for validating Content Dictionaries. The non-normative Chapter 1 of this document briefly overviews the history of OpenMath.


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## Chapter 1

## OpenMath Movement

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Reword to reflect birth of OM Society

This chapter is a historical account of OpenMath and should be regarded as non-normative.
OpenMath is a standard for representing mathematical objects, allowing them to be exchanged between computer programs, stored in databases, or published on the worldwide web. While the original designers were mainly developers of computer algebra systems, it is now attracting interest from other areas of scientific computation and from many publishers of electronic documents with a significant mathematical content. There is a strong relationship to the MathML recommendation [3] from the Worldwide Web Consortium, and a large overlap between the two developer communities. MathML deals principally with the presentation of mathematical objects, while OpenMath is solely concerned with their semantic meaning or content. While MathML does have some limited facilities for dealing with content, it also allows semantic information encoded in OpenMath to be embedded inside a MathML structure. Thus the two technologies may be seen as highly complementary.

### 1.1 History

OpenMath was originally developed through a series of workshops held in Zurich (1993 and 1996), Oxford (1994), Amsterdam (1995), Copenhagen (1995), Bath (1996), Dublin (1996), Nice (1997), Yorktown Heights (1997), Berlin (1998), and Tallahassee (1998). The participants in these workshops formed a global OpenMath community which was coordinated by a Steering Committee and operated through electronic mailing groups and ad-hoc working parties. This loose arrangement has been formalised through the establishment of an OpenMath Society. Up until the end of 1996 much of the work of the community was funded through a grant from the Human Capital and Mobility program of the European Union, the contributions of several institutions and individuals. A document outlining the objectives and basic design of OpenMath was produced (later published as [1]). By the end of 1996 a simplified specification had been agreed on and some prototype implementations have come about [9].

In 1996 a group of European participants in OpenMath decided to bid for funding under the European Union's Fourth Framework Programme for strategic research in information technology. This bid was successful and the project started in late 1997. The principal aims of the project are to formalise OpenMath as a standard and to develop it further through industrial applications; this document is a product of that process and draws heavily on the previous work

[^0]described earlier. OpenMath participants from all over the world continue to meet regularly and cooperate on areas of mutual interest, and recent workshops in Tallahassee (November 1998) and Eindhoven (June 1999) endorsed drafts of this document as the current OpenMath standard.

In November 1998 the OpenMath Society has been established to coordinate all OpenMath

## Chapter 2

## Introduction to OpenMath

This chapter briefly introduces OpenMath concepts and notions that are referred to in the rest of this document.

### 2.1 OpenMath Architecture

The architecture of OpenMath is described in Figure 2.1 and summarizes the interactions among the different OpenMath components. There are three layers of representation of a mathematical object [7]. A private layer that is the internal representation used by an application. An abstract layer that is the representation as an OpenMath object. Third is a communication layer that translates the OpenMath object representation to a stream of bytes. An application dependent program manipulates the mathematical objects using its internal representation, it can convert them to OpenMath objects and communicate them by using the byte stream representation of OpenMath objects.

### 2.2 OpenMath Objects and Encodings

OpenMath objects are representations of mathematical entities that can be communicated among various software applications in a meaningful way, that is, preserving their "semantics".

OpenMath objects and encodings are described in detail in Chapter 3 and Chapter 4.
The standard endorses encodings in XML and binary format. These are the encodings supported by the official OpenMath libraries. However they are not the only possible encodings of OpenMath objects. Users that wish to define their own encoding using some other specific language (e.g. Lisp) may do so provided there is an effective translation of this encoding to an official one.

### 2.3 Content Dictionaries

Content Dictionaries (CDs) are used to assign informal and formal semantics to all symbols used in the OpenMath objects. They define the symbols used to represent concepts arising in a

[^1]Program A


Figure 2.1: The OpenMath Architecture
particular area of mathematics.
The Content Dictionaries are public, they represent the actual common knowledge among OpenMath applications. Content Dictionaries fix the "meaning" of objects independently of the application. The application receiving the object may then recognize whether or not, according to the semantics of the symbols defined in the Content Dictionaries, the object can be transformed to the corresponding internal representation used by the application.

### 2.4 Additional Files

Several additional files are related to Content Dictionaries. Signature files contain the signatures of symbols defined in some OpenMath Content Dictionary and their format is endorsed by this standard.

Furthermore, the standard fixes how to define as a CDGroup a specific set of Content Dictionaries.

Auxiliary files that define presentation and rendering or that are used for manipulating and

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Removed mention to DefMP files processing Content Dictionaries are not discussed by the standard.

### 2.5 Phrasebooks

The conversion of an OpenMath object to/from the internal representation in a software application is performed by an interface program called Phrasebook. The translation is governed by the Content Dictionaries and the specifics of the application. It is envisioned that a software application dealing with a specific area of mathematics declares which Content Dictionaries it understands. As a consequence, it is expected that the Phrasebook of the application is able to translate OpenMath objects built using symbols from these Content Dictionaries to/from the internal mathematical objects of the application.
OpenMath objects do not specify any compuational behaviour, they merely represent mathematical expressions. Part of the OpenMath philosophy is to leave it to the application to decide what it does with an object once it has received it. OpenMath is not a query or programming language. Because of this, OpenMath does not prescribe a way of forcing "evaluation" or "simplification" of objects like $2+3$ or $\sin (\pi)$. Thus, the same object $2+3$ could be transformed to 5 by a computer algebra system, or displayed as $2+3$ by a typesetting tool.

## Chapter 3

## OpenMath Objects

In this chapter we provide a self-contained description of OpenMath objects. We first do so at an informal level (Section 3.2) and next by means of an abstract grammar description (Section 3.1).

### 3.1 Formal Definition of OpenMath Objects

OpenMath represents mathematical objects as terms or as labelled trees that are called OpenMath objects or OpenMath expressions. The definition of an abstract OpenMath object is then the following.

### 3.1.1 Basic OpenMath objects

The Basic OpenMath Objects form the leaves of the OpenMath Object tree. A Basic OpenMath Object is of one of the following.
(i) Integer.

Integers in the mathematical sense, with no predefined range. They are "infinite precision" integers (also called "bignums" in computer algebra).
(ii) IEEE floating point number.

Double precision floating-point numbers following the IEEE 754-1985 standard [11].
(iii) Character string.

A Unicode Character string. This also corresponds to 'characters' in XmL.
(iv) Bytearray.

A sequence of bytes.
(v) Symbol.

A Symbol encodes two fields of information, a name and a Content Dictionary. Each is a sequence of characters matching a regular expression, as described below.
(vi) Variable.

A Variable consists of a name which is a sequence of characters matching a regular expression, as described below.

### 3.1.2 Compound OpenMath Objects

OpenMath objects are built recursively as follows.
(i) Basic OpenMath objects are OpenMath objects.
(ii) If $A_{1}, \ldots, A_{n}(n>0)$ are OpenMath objects, then

$$
\operatorname{application}\left(A_{1}, \ldots, A_{n}\right)
$$

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Add integer and float
is an OpenMath application object.
(iii) If $S_{1}, \ldots, S_{n}$ are OpenMath symbols, and $A, A_{1}, \ldots, A_{n},(n>0)$ are OpenMath objects, then

$$
\operatorname{attribution}\left(A, S_{1} A_{1}, \ldots, S_{n} A_{n}\right)
$$

is an OpenMath attribution object and $A$ is the object stripped of attributions. The operation of recursively applying stripping to the stripped object is called flattening of the attribution. When the stripped object after flattening is a variable, the attributed object is called attributed variable.
(iv) If $B$ and $C$ are OpenMath objects, and $v_{1}, \ldots, v_{n}(n \geq 0)$ are OpenMath variables or attributed variables, then

$$
\operatorname{binding}\left(B, v_{1}, \ldots, v_{n}, C\right)
$$

is an OpenMath binding object.
(v) If $S$ is an OpenMath symbol and $A_{1}, \ldots, A_{n}(n \geq 0)$ are OpenMath objects, then

$$
\operatorname{error}\left(S, A_{1}, \ldots, A_{n}\right)
$$

is an OpenMath error object.

### 3.2 Further Description of OpenMath Objects

Informally, an OpenMath object can be viewed as a tree and is also referred to as a term. The objects at the leaves of OpenMath trees are called basic objects. The basic objects supported by OpenMath are:

Integer Arbitrary Precision integers.
Float OpenMath floats are ieee 754 Double precision floating-point numbers. Other types of floating point number may be encoded in OpenMath by the use of suitable content dictionaries.
Character strings are sequences of characters. These characters come from the Unicode standard [8].
Bytearrays are sequences of bytes. There is no "byte" in OpenMath as an object of its own. However, a single byte can of course be represented by a bytearray of length 1 . The difference between strings and bytearrays is the following: a character string is a sequence of bytes with a fixed interpretation (as characters, Unicode texts may require several bytes to code one character), whereas a bytearray is an uninterpreted sequence of bytes with no intrinsic meaning. Bytearrays could be used inside OpenMath errors to provide information to, for example, a debugger; they could also contain intermediate results of calculations, or 'handles' into computations or databases.

Symbols are uniquely defined by the Content Dictionary in which they occur and by a name. In definition in Section 3.1 we have left this information implicit. However, it should be kept in mind that all symbols appearing in an OpenMath object are defined in a Content Dictionary. The form of these definitions is explained in Chapter 5. Each symbol has no more than one definition in a Content Dictionary. Many Content Dictionaries may define differently a symbol with the same name (e.g., the symbol union is defined as associative-commutativeset theoretic union in a Content Dictionary set1 but another Content Dictionary, multiset1 might define a symbol union as the union of multi-sets. The name of a symbol can only contain alphanumeric characters and underscores. More precisely, a symbol name matches the following regular expression:

$$
[\mathrm{A}-\mathrm{Za}-\mathrm{z}]\left[\mathrm{A}-\mathrm{Za}-\mathrm{zO}-\mathrm{9}_{-}\right]^{*}
$$

Notice that these symbol names are case sensitive. OpenMath recommends that symbol names should be no longer than 100 characters.
Variables are meant to denote parameters, variables or indeterminates (such as bound variables of function definitions, variables in summations and integrals, independent variables of derivatives). Plain variable names are restricted to use a subset of the printable ASCII characters. Formally the names must match the regular expression:

$$
\left[A-Z a-z 0-9=+(),-. /: ?!\# \$ \% * ;=@[]^{\wedge}{ }^{\prime}\{1\}\right]+
$$

The four following constructs can be used to make compound OpenMath objects.
Application constructs an OpenMath object from a sequence of one or more OpenMath objects. The first argument of application is referred to as "head" while the remaining objects are called "arguments". An OpenMath application object can be used to convey the mathematical notion of application of a function to a set of arguments. For instance, suppose that the OpenMath symbol sin is defined in a Content Dictionary for trigonometry, then application $(\sin , x)$ is the abstract OpenMath object corresponding to $\sin (x)$. More generally, an OpenMath application object can be used as a constructor to convey a mathematical object built from other objects such as a polynomial constructed from a set of monomials. Constructors build inhabitants of some symbolic type, for instance the type of rational numbers or the type of polynomials. The rational number, usually denoted as $1 / 2$, is represented by the OpenMath application object application(Rational, 1, 2). The symbol Rational must be defined, by a Content Dictionary, as a constructor symbol for the rational numbers.
Binding objects are constructed from an OpenMath object, and from a sequence of zero or more variables followed by another OpenMath object. The first OpenMath object is the "binder" object. Arguments 2 to $n-1$ are always variables to be bound in the "body" which is the $n^{t h}$ argument object. It is allowed to have no bound variables, but the binder object and the body should be present. Binding can be used to express functions or logical statements. The function $\lambda x \cdot x+2$, in which the variable $x$ is bound by $\lambda$, corresponds to a binding object having as binder the OpenMath symbol lambda:

$$
\text { binding(lambda, } x \text {, application(plus, } x, 2) \text { ). }
$$

Binding of several variables as in:

$$
\operatorname{binding}\left(B, v_{1}, \ldots, v_{n}, C\right)
$$



Figure 3.1: The OpenMath application and binding objects for $\sin (x)$ and $\lambda x . x+2$ in tree-like notation.
is semantically equivalent to composition of binding of a single variable, namely

$$
\operatorname{binding}\left(B, v_{1},\left(\operatorname { b i n d i n g } \left(B, v_{2},\left(\ldots, \operatorname{binding}\left(B, v_{n}, C\right) \ldots\right) .\right.\right.\right.
$$

Note that it follows from this that repeated occurences of the same variable in a binding operator are allowed. For example the object

$$
\operatorname{binding}(\text { lambda, } v, v, \text { application }(\text { times }, v, v))
$$

is semantically equivalent to:

$$
\text { binding(lambda, } v, \operatorname{binding}(\operatorname{lambda}, v, \text { application }(\operatorname{times}, v, v)))
$$

so that the outermost binding is actually a constant function ( $v$ does not occur free in the body application(times, $v, v)$ ))).
Phrasebooks are allowed to use $\alpha$ conversion in order to avoid clashes of variable names. Suppose an object $\Omega$ contains an occurrence of the object binding $(B, v, C)$. This object $\operatorname{binding}(B, v, C)$ can be replaced in $\Omega$ by $\operatorname{binding}\left(B, z, C^{\prime}\right)$ where $z$ is a variable not occurring free in $C$ and $C^{\prime}$ is obtained from $C$ by replacing each free (i.e., not bound by any intermediate binding construct) occurrence of $v$ by $z$. This operation preserves the semantics of the object $\Omega$. In the above example, a phrasebook is thus allowed to transform the object to, e.g.

$$
\text { binding(lambda, } v, \text { binding(lambda, } z, \text { application(times, } z, z)) \text { ). }
$$

Attribution decorates an object with a sequence of one or more pairs made up of an OpenMath symbol, the "attribute", and an associated OpenMath object, the "value of the attribute". The value of the attribute can be an attribution object itself. As example of this, consider the OpenMath objects representing groups, automorphism groups, and group dimensions. It is then possible to attribute an OpenMath object representing a group by its automorphism group, itself attributed by its dimension.
Composition of attributions, as in

$$
\text { attribution(attribution } \left.\left(A, S_{1} A_{1}, \ldots, S_{h} A_{h}\right), S_{h+1} A_{h+1}, \ldots, S_{n} A_{n}\right)
$$

is semantically equivalent to a single attribution, that is

$$
\operatorname{attribution}\left(A, S_{1} A_{1}, \ldots, S_{h} A_{h}, S_{h+1} A_{h+1}, \ldots, S_{n} A_{n}\right)
$$

The operation that produces an object with a single layer of attribution is called flattening.
Multiple attributes with the same name are allowed. While the order of the given attributes does not imply any notion of priority, potentially it could be significant. For instance, consider the case in which $S_{h}=S_{n}(h<n)$ in the example above. Then, the object is to be interpreted as if the value $A_{n}$ overwrites the value $A_{h}$. (OpenMath however does not mandate that an application preserves the attributes or their order.)
Objects can be decorated in a multitude of ways. In [4], typing of OpenMath objects is expressed by using an attribution. The object attribution( $A$, type $t$ ) represents the judgment stating that object $A$ has type $t$. Note that both $A$ and $t$ are OpenMath objects.
Attribution can act as either annotation, in the sense of adornment, or as modifier. In the former case, replacement of the adorned object by the object itself is probably not harmful (preserves the semantics). In the latter case however, it may very well be. Therefore, attribution in general should by default be treated as a construct rather than as adornment. Only when the CD definitions of the attributes make it clear that they are adornments, can the attributed object be viewed as semantically equivalent to the stripped object.
Error is made up of an OpenMath symbol and a sequence of zero or more OpenMath objects. This object has no direct mathematical meaning. Errors occur as the result of some treatment on an OpenMath object and are thus of real interest only when some sort of communication is taking place. Errors may occur inside other objects and also inside other errors. Error objects might consist only of a symbol as in the object: error $(S)$.

### 3.3 Summary

- OpenMath supports basic objects like integers, symbols, floating-point numbers, character strings, bytearrays, and variables.
- OpenMath compound objects are of four kinds: applications, bindings, errors, and attributions.
- OpenMath objects have the expressive power to cover all areas of computational mathematics.

Observe that an OpenMath application object is viewed as a "tree" by software applications that do not understand Content Dictionaries, whereas a Phrasebook that understands the semantics of the symbols, as defined in the Content Dictionaries, should interpret the object as functional application, constructor, or binding accordingly. Thus, for example, for some applications, the OpenMath object corresponding to $2+5$ may result in a command that writes 7 .

## Chapter 4

## OpenMath Encodings

In this chapter, two encodings are defined that map between OpenMath objects and byte streams. These byte streams constitute a low level representation that can be easily exchanged between processes (via almost any communication method) or stored and retrieved from files.

The first encoding uses ISO 646:1983 characters [12] (also known as ASCII characters) and is an XmL application. Although the XML markup of the encoding uses only ASCII characters, OpenMath strings may uses arbitrary Unicode/ISO 10646:1988 characters [8] It can be used, for example, to send OpenMath objects via e-mail, news, cut-and-paste, etc. The texts produced by this encoding can be part of XML documents.

The second encoding is a binary encoding that is meant to be used when the compactness of the encoding is important (interprocess communications over a network is an example).

Note that these two encodings are sufficiently different for autodetection to be effective: an application reading the bytes can very easily determine which encoding is used.

### 4.1 The xml Encoding

This encoding has been designed with two main goals in mind:

1. to provide an encoding that uses the most common character set (so that it can be easily included in most documents and transport protocols) and that is both readable and writable by a human.
2. to provide an encoding that can be included (embedded) in XML documents.

### 4.1.1 A Grammar for the xml Encoding

The xmL encoding of an OpenMath object is defined by the DTD given in Figure 4.1 below, with the following additional rules not implied by the XML DTD.

- Comments are permitted only between elements, not within element character data.
- Processing Instructions are only allowed before the OMOBJ element.
- The content of an OMB element, is a valid base64-encoded text.
- The character data forming element content and attribute values matches the regular expressions of Figure 4.2.

In addition, if the xML document encoding the OpenMath object is linearised into the XML concrete syntax, the following further constraints apply, which ensure thet the encoding may be read by OpenMath applications that may not include a full xmL parser.

- The document should use UTF-8 encoding.
- Entity and character references should not be used.
- A <!DOCTYPE declaration should not be used.
- The xmL empty element form <.../> should always be used to encode elements such as OMF which are specified in the DTD as being EMPTY. It should never be used for elements that may sometimes be empty, such as OMSTR.

Such a linearisation of an XmL encoded OpenMath Object would match the match the character based grammar given in Figure 4.2.

The notation used in this section and in Figure 4.2 should be quite straightforward (+ meaning "one or more", ? meaning zero or one, and | meaning "or"). The start symbol of the grammar is "start", "space" stands for the space character, "cr" for the carriage return character, "nl" for the line feed character and "tab" for the horizontal tabulation character.

### 4.1.2 Description of the Grammar

An encoded OpenMath object is placed inside an OMOBJ element. This element can contain the elements (and integers) as described above.

We briefly discuss the xmL encoding for each type of OpenMath object starting from the basic objects.

Integers are encoded using the OMI element around the sequence of their digits in base 10 or 16 (most significant digit first). White space may be inserted between the characters of the integer representation, this will be ignored. After ignoring white space, integers written in base 10 match the regular expression -? [0-9]+. Integers written in base 16 match - ? $x[0-9 A-F]+$.

The integer 10 can be thus encoded as <OMI> 10 </OMI> or as <OMI> xA </OMI> but neither <OMI> +10 </OMI> nor <OMI> +xA </OMI> can be used.
The negative integer -120 can be encoded as either as decimal <OMI> -120 </OMI> or as hexadecimal <0MI> -x78 </OMI>.
Symbols are encoded using the OMS element. This element has two XmL-attributes cd and name. The value of cd is the name of the Content Dictionary in which the symbol is defined and the value of name is the name of the symbol. The name of the Content Dictionary is compulsory, but a future revision of the OpenMath standard might introduce a defaulting mechanism. For example, <OMS cd="transc" name="sin"/> is the encoding of the symbol named $\sin$ in the Content Dictionary named transc.

```
<!-- DTD for OM Objects - sb 29.10.98 -->
<!-- sb 3.2.99 -->
<!-- general list of embeddable elements
    : excludes OMATP as this is only embeddable in OMATTR
    : excludes OMBVAR as this is only embeddable in OMBIND -->
<!ENTITY % omel "OMS | OMV | OMI | OMB | OMSTR
                | OMF | OMA | OMBIND | OME
                        | OMATTR ">
<!-- things which can be variables -->
<!ENTITY % omvar "OMV | OMATTR" >
<!-- symbol -->
<!ELEMENT OMS EMPTY>
<!ATTLIST OMS name CDATA #REQUIRED cd CDATA #REQUIRED >
<!-- variable -->
<!ELEMENT OMV EMPTY>
<!ATTLIST OMV name CDATA #REQUIRED >
<!-- integer -->
<!ELEMENT OMI (#PCDATA) >
<!-- byte array -->
<!ELEMENT OMB (#PCDATA) >
<!-- string -->
<!ELEMENT OMSTR (#PCDATA) >
<!-- floating point -->
<!ELEMENT OMF EMPTY>
<!ATTLIST OMF dec CDATA #IMPLIED hex CDATA #IMPLIED>
<!-- apply constructor -->
<!ELEMENT OMA (%omel;)+ >
<!-- binding constructor & variable -->
<!ELEMENT OMBIND ((%omel;), OMBVAR, (%omel;)) >
<!ELEMENT OMBVAR (%omvar;)+ >
<!-- error -->
<!ELEMENT OME (OMS, (%omel;)*) >
<!-- attribution constructor & attribute pair constructor -->
<!ELEMENT OMATTR (OMATP, (%omel;)) >
<!ELEMENT OMATP (OMS, (%omel;))+ >
<!-- OM object constructor -->
<!ELEMENT OMOBJ (%omel;) >
<!ATTLIST OMOBJ xmlns CDATA #FIXED "http://www.openmath.org/OpenMath">
```

Figure 4.1: DTD for the OpenMath XML encoding of objects.

```
S loce(space|tab|cr|nl)+
cdname }\longrightarrow [a-z][a-z0-9_]***
symbname \longrightarrow [A-Za-z][A-Za-zO-9_]*
fpdec }\longrightarrow\quad(-?)([0-9]+)?(.[0-9]+)?(e([+|-]?)[0-9]+)
fphex }\longrightarrow\quad[0-9ABCDEF]
varname \longrightarrow ([A-Za-z0-9+=(),-./:?!##%*;@[]^_`{1}])+
base64 \longrightarrow ([A-Za-z0-9+/=]|S)+
char }\longrightarrow\quadXML Character Data
```



Figure 4.2: Grammar for the XML encoding of OpenMath objects.

Variables are encoded using the OMV element, with only one XML-attribute, name, whose value is the variable name. The variable name is a subset of the printable ASCII set of characters. In particular, neither spaces nor double-quote " are allowed in variable names. For instance, the encoding of the object representing the variable $x$ is: <OMV name="x"/>
Floating-point numbers are encoded using the OMF element that has either the xML-attribute dec or the XML-attribute hex. The two XML-attributes cannot be present simultaneously. The value of dec is the floating-point number expressed in base 10 , using the common syntax:

$$
(-?)([0-9]+) ?(" \cdot "[0-9]+) ?(e(-?)[0-9]+) ?
$$

The value of hex is the digits of the floating-point number expressed in base 16, with digits $0-9$, A-F (mantissa, exponent, and sign from lowest to highest bits) using a least significant byte ordering. For example, <OMF dec="1.0e-10"/> is a valid floating-point number.
Character strings are encoded using the OMSTR element. Its content is a Unicode text (The default encoding is UTF-8[17], although XML encoded OpenMath may be embedded in a containing XmL document that specifies alternative encoding in the xmbdeclaration. Note that as always in XML the characters < and \& need to be represented by the entity references \< and \& respectively.
Bytearrays are encoded using the OMB element. Its content is a sequence of characters that is a base64 encoding of the data. The base64 encoding is defined in RFC 1521 [2]. Basically, it represents an arbitrary sequence of octets using 64 "digits" (A through Z , a through $\mathbf{z}$, 0 through $9,+$ and $/$, in order of increasing value). Three octets are represented as four digits (the = character for padding to the right at the end of the data). All line breaks and carriage return, space, form feed and horizontal tabulation characters are ignored. The reader is refered to [2] for more detailed information.

In detail the encoding of an OpenMath object is described below.

Applications are encoded using the OMA element. The application whose root is the OpenMath object $e_{0}$ and whose arguments are the OpenMath objects $e_{1}, \ldots, e_{n}$ is encoded as <OMA> $C_{0} C_{1} \ldots C_{n}$ </OMA> where $C_{i}$ is the encoding of $e_{i}$.

For example, application( $\sin , x)$ is encoded as:

```
<OMA>
<OMS cd="transc1" name="sin"/>
<OMV name="x"/>
</OMA>
```

provided that the symbol sin is defined to be a function symbol in a Content Dictionary named transc1.
Binding is encoded using the OMBIND element. The binding by the OpenMath object $b$ of the OpenMath variables $x_{1}, x_{2}, \ldots, x_{n}$ in the object $c$ is encoded as <OMBIND> $B$ <OMBVAR> $X_{1} \ldots X_{n}$ </OMBVAR> $C$ </OMBIND> where $B, C$, and $X_{i}$ are the encodings of $b, c$ and $x_{i}$, respectively.
For instance the encoding of binding(lambda, $x$, application $(\sin , x)$ ) is:

```
<OMBIND>
    <OMS cd="fns1" name="lambda"/>
    <OMBVAR>
            <OMV name="x"/>
    </OMBVAR>
    <OMA>
            <OMS cd="transc1" name="sin"/>
            <OMV name="x"/>
    </OMA>
</OMBIND>
```

Binders are defined in Content Dictionaries, in particular, the symbol lambda is defined in the Content Dictionary fns 1 for functions over functions.
Attributions are encoded using the OMATTR element. If the OpenMath object $e$ is attributed with $\left(s_{1}, e_{1}\right), \ldots,\left(s_{n}, e_{n}\right)$ pairs (where $s_{i}$ are the attributes), it is encoded as <OMATTR> <OMATP> $S_{1} C_{1} \ldots S_{n} C_{n}$ </OMATP> $E$ </OMATTR> where $S_{i}$ is the encoding of the symbol $s_{i}, C_{i}$ of the object $e_{i}$ and $E$ is the encoding of $e$.
Examples are the use of attribution to decorate a group by its automorphism group:

```
<OMATTR>
    <OMATP>
                <OMS cd="groups" name="automorphism_group" />
                [..group-encoding..]
    </OMATP>
    [..group-encoding..]
</OMATTR>
```

or to express the type of a variable:

```
<OMATTR>
    <OMATP>
                <OMS cd="ecc" name="type" />
                <OMS cd="ecc" name="real" />
            </OMATP>
            <OMV name="x" />
</OMATTR>
```

Errors are encoded using the OME element. The error whose symbol is $s$ and whose arguments are the OpenMath objects $e_{1}, \ldots, e_{n}$ is encoded as <OME> $C_{s} C_{1} \ldots C_{n}$ </OME> where $C_{s}$ is the encoding of $s$ and $C_{i}$ the encoding of $e_{i}$.
If an aritherror Content Dictionary contained a DivisionByZero symbol, then the object error(DivisionByZero, application(divide, $x, 0$ )) would be encoded as follows:

```
<OME>
<OMS cd="aritherror" name="DivisionByZero"/>
<OMA>
        <OMS cd="arith1" name="divide" />
        <OMV name="x"/>
        <OMI> 0 </OMI>
</OMA>
</OME>
```

1999/09/21
New section on embedding OM in XML documents

2000/03/20
Namespace URI, as discussed on OM Soc list

1999/06/24
New attrvar production

### 4.1.3 Embedding OpenMath in XML Documents

The above encoding of xmL encoded OpenMath specifies the grammar to be used in files that encode a single OpenMath object, and specifies the character streams that a conforming OpenMath application should be able to accept or produce.
When embedding XmL encoded OpenMath objects into a larger XML document one may wish, or need, to use other XML features. For example use of extra XML attributes to specify xmL Namespaces [16] or xml:lang attributes to specify the language used in strings [14]. Also, the encoding used in the larger document may not be UTF-8.
In particular, if OpenMath is used with applications that use the XML Namespace Recommnedation [16] then they should ensure that OpenMath elements are in the namespace http: www. openmath.org/OpenMath. This is most conveniently achieved by adding the namespace declaration
xmlns="http:www.openmath.org/OpenMath"
as an attribute to each OMOBJ element in the document.
If such XmL features are used then the XML application controlling the document must, if passing the OpenMath fragment to an OpenMath application, remove any such extra attributes and must ensure that the fragment is encoded according to the grammar specified above.

### 4.2 The Binary Encoding

The binary encoding was essentially designed to be more compact than the XmL encodings, so that it can be more efficient if large amounts of data are involved. For the current encoding, we tried to keep the right balance between compactness, speed of encoding and decoding and simplicity (to allow a simple specification and easy implementations).

### 4.2.1 A Grammar for the Binary Encoding

Figure 4.3 gives a grammar for the binary encoding. The following conventions are used in this section: $[n]$ denotes a byte whose value is the integer $n$ ( $n$ can range from 0 to 255 ), $\{m\}$ denotes four bytes representing the (unsigned) integer $m$ in network byte order, [-] denotes an arbitrary byte, $\{-\}$ denotes an arbitrary sequence of four bytes. name:n denotes a sequence of $n$ bytes named name. name: $2 n$ denotes a sequence of $2 n$ bytes. "start" is the start symbol of the grammar.

### 4.2.2 Description of the Grammar

An OpenMath object is encoded as a sequence of bytes starting with the begin object tag (value 24) and ending with the end object tag (value 25). These are similar to the <0MOBJ> and </OMOBJ> tags of the XML encoding.

The encoding of each kind of OpenMath object begins with a tag that is a single byte, holding a token identifier and two flags, the long flag and the shared flag. The identifier is stored in the first 6 bits ( 1 to 6 ). The long flag is the eighth bit and the shared flag is the seventh bit.

Here is a description of the binary encodings of every kind of OpenMath object:.

| start object | $\longrightarrow$ | [24] object [25] |
| :---: | :---: | :---: |
|  | $\longrightarrow$ | integer |
|  | \| | float |
|  | \| | variable |
|  | \| | symbol |
|  | \| | string |
|  | \| | bytearray |
|  | \| | construct |
| integer | $\longrightarrow$ | [1] [-] |
|  |  | [1+128] ${ }_{\text {- }}$ \} |
|  | \| | [2] [n] [-] digits:n |
|  | \| | [2+128] n$\}$ [-] digits:n |
| float variable | $\longrightarrow$ | [3] \{-\} \{-\} |
|  | $\longrightarrow$ | [5] [n] varname:n |
|  | \| | [ $5+128]\{\mathrm{n}\}$ varname:n |
|  | \| | $[5+64][\mathrm{n}]$ |
| symbol | $\xrightarrow{\longrightarrow}$ | $\begin{aligned} & {[8][\mathrm{n}][\mathrm{m}] \text { cdname:n symbname:m }} \\ & \begin{array}{l} {[8+128]\{\mathrm{n}\}\{\mathrm{m}\}} \\ {[8+64][\mathrm{n}]} \end{array} \end{aligned}$ |
| string | $\longrightarrow$ | $[6][n]$ chars:n |
|  | \| | [6+128] n \} chars:n |
|  |  | [7] [n] chars:2n |
|  | \| | [ $7+128]\{\mathrm{n}\}$ chars:2n |
|  | \| | [7+64][n] |
| bytearray | $\longrightarrow$ | [4] [n] bytes:n |
|  | \| | [4 +128$]\{\mathrm{n}\}$ bytes:n |
| construct | $\longrightarrow$ | [16] object objects [17] |
|  |  | [22] symbol objects [23] |
|  | \| | [18] attrpairs object [19] |
|  | \| | [26] object bvars object [27] |
| attrpairs | $\longrightarrow$ | [20] pairs [21] |
| pairs | $\longrightarrow$ | symbol object |
|  | \| | symbol object pairs |
| bvars | $\longrightarrow$ | [28] vars [29] |
| vars | $\longrightarrow$ | attrvar |
|  | \| | attrvar vars |
| attrvar | $\longrightarrow$ | variable |
|  | 1 | [18] attrpairs attrvar [19] |
| objects | $\longrightarrow$ |  |
|  | , | object objects |

Figure 4.3: Grammar of the binary encoding of OpenMath objects.

Integers are encoded depending on how large they are. There are four possible formats. Integers between - 128 and 127 are encoded as the small integer tag (1) followed by a single byte that is the value of the integer (interpreted as a signed character). For example 16 is encoded as $0 x 010 x 10$. Integers between $-2^{31}(-2147483648)$ and $2^{31}-1(2147483647)$ are encoded as the small integer tag with the long flag set followed by the integer encoded in little endian format on four bytes (network byte order: the most significant byte comes first). For example, 128 is encoded as $0 \times 810 \times 00000080$. The most general encoding begins with the big integer tag (token identifier 2) with the long flag set if the number of bytes in the encoding of the digits is greater or equal than 256. It is followed by the length (in bytes) of the sequence of digits, encoded on one byte ( 0 to 255 , if the long flag was not set) or four bytes (network byte order, if the long flag was set). It is then followed by a byte describing the sign and the base. This 'sign/base' byte is $+(0 \times 2 B)$ or $-(0 x 2 D)$ for the sign ored with the base mask bits that can be 0 for base 10 or 0 x 40 for base 16 . It is followed by the strings of digits (as characters) in their natural order (as in the xml encoding). For example, $8589934592\left(2^{33}\right)$ is encoded $0 \times 020 \times 0$ A $0 x 2 \mathrm{~B} 0 \times 38353839393334353932$ and xfffffff1 is encoded as $0 x 020 x 08$ 0x6b $0 x 6666666666666631$. Note that it is permitted to encode a "small" integer in any "bigger" format.
Symbols are encoded as the symbol tag (8) with the long flag set if the maximum of the length of the Content Dictionary name and the symbol name is greater than or equal to 256 (note that this should never be the case if the rules on symbols and Content Dictionary names are applied), then followed by the length of the Content Dictionary name as a byte (if the long flag was not set) or a four byte integer (in network byte order) followed by the length of the symbol name as a byte (if the long flag was not set) or a four byte integer (in network byte order), followed by the characters of the Content Dictionary name, followed by the characters of the symbol name.
Variables are encoded using the variable tag (5) with the long flag set if the number of bytes (characters) in the variable name is greater than or equal to 256 (this should never happen if the rules on variables are followed). Then, there is the number of characters as a byte (if the long flag was not set) or a four byte integer (in network byte order), followed by the characters of the name of the variable. For example, the variable x is encoded as 0 x 05 $0 \times 010 x 78$.
Floating-point number are encoded using the floating-point number tag (3) followed by eight bytes that are the IEEE 754 representation [11], most significant bytes first. For example, 0.1 is encoded as $0 x 030 x 000000000000 f 03 f$.

Character string are encoded in two ways depending on whether the string contains UTF-16 characters or not. If the string contains only 8 bit characters, it is encoded as the one byte character string tag (6) with the long flag set if the number of bytes (characters) in the string is greater than or equal to 256 . Then, there is the number of characters as a byte (if the length flag was not set) or a four byte integer (in network byte order), followed by the characters in the string. If the string contains two byte characters, it is encoded as the two byte character string tag (7) with the long flag set if the number of characters in the string is greater or equal to 256 . Then, there is the number of characters as a byte (if the long flag was not set) or a four byte integer (in network byte order), followed by the characters (UTF-16 encoded Unicode).
Bytearrays are encoded using the bytearray tag (4) with the long flag set if the number of bytes in the number of elements is greater than or equal to 256 . Then, there is the number of elements, as a byte (if the long flag was not set) or a four byte integer (in network byte order), followed by the elements of the arrays in their normal order.

Applications are encoded using the application tag (16). More precisely, the application of $E_{0}$ to $E_{1} \ldots E_{n}$ is encoded using the application tag (16), the sequence of the encodings of $E_{0}$ to $E_{n}$ and the end application tag (17).
Bindings are encoded using the binding tag (26). More precisely, the binding by $B$ of variables $V_{1} \ldots V_{n}$ in $C$ is encoded as the binding tag (26), followed by the encoding of $B$, followed by the binding variables tag (28), followed by the encodings of the variables $V_{1} \ldots V_{n}$, followed by the end binding variables tag (29), followed by the encoding of $C$, followed by the end binding tag (27).
Attribution are encoded using the attribution tag (18). More precisely, attribution of the object $E$ with $\left(S_{1}, E_{1}\right), \ldots\left(S_{n}, E_{n}\right)$ pairs (where $S_{i}$ are the attributes) is encoded as the attributed object tag (18), followed by the encoding of the attribute pairs as the attribute pairs tag (20), followed by the encoding of each symbol and value, followed by the end attribute pairs tag (21), followed by the encoding of $E$, followed by the end attributed object tag (19).
Error are encoded using the error tag (22). More precisely, $S_{0}$ applied to $E_{1} \ldots E_{n}$ is encoded as the error tag (22), the encoding of $S_{0}$, the sequence of the encodings of $E_{0}$ to $E_{n}$ and the end error tag (23).

### 4.2.2.1 Sharing

This binary encoding supports the sharing of symbols, variables and strings (up to a certain length for strings) within one object. That is, sharing between objects is not supported. A reference to a shared symbol, variable or string is encoded as the corresponding tag with the long flag not set and the shared flag set, followed by a positive integer $n$ coded on one byte ( 0 to 255 ). This integer references the $n+1$-th such sharable sub-object (symbol, variable or string up to 255 characters) in the current OpenMath object (counted in the order they are generated by the encoding). For example, $0 \times 480 x 01$ references a symbol that is identical to the second symbol that was found in the current object. Strings with 8 bit characters and strings with 16 bit characters are two different kinds of objects for this sharing. Only strings containing less than 256 characters can be shared (i.e. only strings up to 255 characters).

### 4.2.3 Implementation Note

A typical implementation of the binary encoding uses four tables, each of 256 entries, for symbol, variables, 8 bit character strings whose lengths are less than 256 characters and 16 bit character strings whose lengths are less than 256 characters. When an object is read, all the tables are first flushed. Each time a sharable sub-object is read, it is entered in the corresponding table if it is not full. When a reference to the shared i-th object of a given type is read, it stands for the i-th entry in the corresponding table. It is an encoding error if the i-th position in the table has not already been assigned (i.e. forward references are not allowed). Sharing is not mandatory, there may be duplicate entries in the tables (if the application that wrote the object chose not to share optimally).
Writing an object is simple. The tables are first flushed. Each time a sharable sub-object is encountered (in the natural order of output given by the encoding), it is either entered in the corresponding table (if it is not full) and output in the normal way or replaced by the right reference if it is already present in the table.

### 4.2.4 Example of Binary Encoding

As an example of this binary encoding, we can consider the OpenMath object whose xmL encoding is

```
<0MOBJ>
    <OMA>
        <OMS name="times" cd="arith1"/>
        <OMA>
            <OMS name="plus" cd="arith1"/>
            <OMV name="x"/>
            <OMV name="y"/>
        </OMA>
        <OMA>
            <OMS name="plus" cd="arith1"/>
            <OMV name="x"/>
            <OMV name="z"/>
        </OMA>
    </OMA>
</OMOBJ>
```

It is binary encoded as the sequence of bytes given by the following table.

| Hex | Meaning | Hex | Meaning |
| :--- | :--- | :--- | :--- |
| 18 | begin object tag | 68 | h .) |
| 10 | begin application tag | 31 | 1.$)$ |
| 08 | symbol tag | 70 | p (symbol name begin |
| 06 | cd length | 6 c | 1. |
| 05 | name length | 75 | u . |
| 61 | a (cd name begin | 73 | s.) |
| 72 | r. | 05 | variable tag |
| 69 | i . | 01 | name length |
| 74 | t. | 78 | x (name) |
| 68 | h . | 05 | variable tag |
| 31 | 1. ) | 01 | name length |
| 74 | t (symbol name begin | 79 | y (variable name) |
| 69 | i . | 11 | end application tag |
| $6 d$ | m. | 10 | begin application tag |
| 65 | e . | 48 | symbol tag (with share bit on) |
| 73 | s .) | 01 | reference to second symbol seen (arith1:plus) |
| 10 | begin application tag | 45 | variable tag (with share bit on) |
| 08 | symbol tag | 00 | reference to first variable seen (x) |
| 06 | cd length | 05 | variable tag |
| 04 | name length | 01 | name length |
| 61 | a (cd name begin | $7 a$ | z (variable name) |
| 72 | r . | 11 | end application tag |
| 69 | i . | 11 | end application tag |
| 74 | t. | 19 | end object tag |

### 4.3 Summary

The key points of this chapter are:

- The xml encoding for OpenMath objects uses most common character sets.
- The xml encoding is readable, writable and can be embedded in most documents and transport protocols.
- The binary encoding for OpenMath objects should be used when efficiency is a key issue. It is compact yet simple enough to allow fast encoding and decoding of objects.


## Chapter 5

## Content Dictionaries

In this chapter we give a brief overview of Content Dictionaries before explicitly stating their functionality and encoding.

### 5.1 Introduction

Content Dictionaries (CDs) are central to the OpenMath philosophy of transmitting mathematical information. It is the OpenMath Content Dictionaries which actually hold the meanings of the objects being transmitted.

For example if application $A$ is talking to application $B$, and sends, say, an equation involving multiplication of matrices, then $A$ and $B$ must agree on what a matrix is, and on what matrix multiplication is, and even on what constitutes an equation. All this information is held within some Content Dictionaries which both applications agree upon.

A Content Dictionary holds the meanings of (various) mathematical "words". These words are OpenMath basic objects referred to as symbols in Section 3.1.

With a set of symbol definitions (perhaps from several content Dictionaries), $A$ and $B$ can now talk in a common "language".
It is important to stress that it is not Content Dictionaries themselves which are being passed, but some "mathematics" whose definitions are held within the Content Dictionaries. This means that the applications must have already agreed on a set of Content Dictionaries which they

Rephrase slightly "understand" (i.e., can cope with to some degree).

In many cases, the Content Dictionaries that an application understands will be constant, and be intrinsic to the application's mathematical use. However the above approach can also be used for applications which can handle every Content Dictionary (such as an OpenMath parser, or perhaps a typesetting system), or alternatively for applications which understand a changeable number of Content Dictionaries (perhaps after being sent Content Dictionaries in some way).

The primary use of Content Dictionaries is thought to be for designers of Phrasebooks,the programs which translate between the OpenMath mathematical object and the corresponding (often internal) structure of the particular application in question. For such a use the Content Dictionaries have themselves been designed to be as readable and precise as possible.

Another possible use for OpenMath Content Dictionaries could rely on their automatic comprehension by a machine (e.g., when given definitions of objects defined in terms of previously understood ones), in which case Content Dictionaries may have to be passed as data. Towards this end, a Content Dictionary has been written which contains a set of symbols sufficient to represent any other Content Dictionary. This means that Content Dictionaries may be passed in the same way as other (OpenMath) mathematical data.

Finally, the syntax of the Content Dictionaries has been designed to be relatively easy to learn and to write, and also free from the need for any specialist software. This is because it is acknowledged that there is an enormous amount of mathematical information to represent, and so most of the Content Dictionaries will be written by "ordinary" mathematicians, encoding their particular fields of expertise. A further reason is that the mathematics conveyed by a specific Content Dictionary should be understandable independently of any application.

The key points from this section are:

- Content Dictionaries should be readable and precise to help Phrasebook designers,
- Content Dictionaries should be readily write-able to encourage widespread use,
- It ought to be possible for a machine to understand a Content Dictionary to some degree.


### 5.2 Content Dictionaries

In this section we define the overall structure of Content Dictionaries.
Other than Content Dictionary comments (which have no real semantics), Content Dictionaries have been designed to hold two types of information: that which is pertinent to the whole Content Dictionary, and that which is restricted to a particular symbol definition. Specific information pertaining to the symbols like the signature and the defining mathematical properties is conveyed in additional files associated to Content Dictionaries.

Information that is pertinent to the whole Content Dictionary includes:

- The name of the Content Dictionary.
- A description of the Content Dictionary.
- A date when the Content Dictionary is next planned to be reviewed.
- A date on which the Content Dictionary was last edited.
- The current version and revision numbers of the Content Dictionary.
- The status of the Content Dictionary.
- An optional URL for this Content Dictionary.
- An optional list of Content Dictionaries on which this Content Dictionary depends. That is, those named in Examples and FMP in this Content Dictionary.
- An optional comment, possibly containing the author's name.

Information that is restricted to a particular symbol includes:

- The name of the symbol.
- A description of this symbol.
- An optional comment.

1999/08/24
More motivation on design of CDs

1999/08/24
New paragraph to reflect recent changes

- Optional properties that this symbol should obey.
- Optional examples of the use of this symbol.

1999/08/24
removed refs to old changes
1999/06/22
new paragraph
1999/08/24
Defmp added
1999/10/04
Rephrase slightly

2000/04/10 MathML 2

1999/06/20
now we have this numbering mechanism, should it be documented?

As mentioned earlier, certain kinds of data pertaining to symbols may be conveyed in files other than a Content Dictionary. In particular, information on signatures according to a type system may be described in Signature Files whose format is given in Section 5.4.1. Other information such as presentation forms, extra defining mathematical properties may be associated with Content Dictionaries using files whose format is not specified by this standard. It is expected that a common method of defining the presentation for OpenMath symbols is via XsL [15] stylesheets giving transformations to MathML.

Content Dictionaries may be grouped into CD Groups. These groups allow applications to easily refer to collections of Content Dictionaries. One particular CDGroup of interest is the "MathML CDGroup". This group expresses the collection of the core Content Dictionaries that is designed to have the same semantic scope as the content elements of MathML 2 [13]. OpenMath objects built from symbols that come from Content Dictionaries in this CDGroup may be expected to be eaily transformed between OpenMath and MathML encodings. The detailed structure of a CDGroup is described in section 5.4.2 below.

### 5.3 The XML Encoding for Content Dictionaries

Content Dictionaries are XML documents. A valid Content Dictionary document should

- be valid according to the DTD given in Figure 5.1,
- adhere to the extra conditions on the content of the elements given in Section 5.3.2.

An example of a complete Content Dictionary is given in Appendix A.1, which is the Meta Content Dictionary for describing Content Dictionaries themselves. A more typical Content Dictionary is given in Appendix A.2, the arith1 Content Dictionary for basic arithmetic functions.

### 5.3.1 The DTD Specification of Content Dictionaries

The XML DTD for Content Dictionaries is given in Figure 5.1. The allowed elements are further described in the following section.

### 5.3.2 Further Requirements of an OpenMath Content Dictionary

The notion of being a valid Content Dictionary is stronger than merely being successfully parsed by the DTD. This is because the content of the elements, referred to in Figure 5.1 as PCDATA and CDATA, must actually make sense to, say, a Phrasebook designer. In this section we define exactly the format of the elements used in Content Dictionaries.

CDName The text occurring in the CDName element corresponds to the name of Content Dictionary, and is of the form specified in Chapter 4.

```
<!-- omcd.dtd -->
<!-- ********************************************************** -->
<!-- -->
<!-- DTD for OpenMath CD -->
<!-- (c) EP24969 the ESPRIT OpenMath Consortium -->
<!-- date = 28.aug.1998 -->
<!-- author = s.buswell sb@stilo.demon.co.uk -->
<!-- -->
<!-- edited by n.howgrave-graham 30.aug.98 -->
<!-- edited by sb 4.sep.98 -->
<!-- edited by nh-g 4.sep.98 -->
<!-- edited by sb 1.nov.98 -->
<!-- edited by dpc 1999-04-13 -->
<!-- edited by dpc 1999-05-11 CDDate & CDVersion -->
<!-- edited by dpc 1999-06-21 Delete Signature&Presentation -->
<!-- Force Name as first child of -->
<!-- CDDefinition -->
<!-- -->
<!-- *********************************************************** -->
<!ELEMENT CDName (#PCDATA) >
<!ELEMENT Description (#PCDATA) >
<!ELEMENT CDReviewDate (#PCDATA) >
<!ELEMENT CDDate (#PCDATA) >
<!ELEMENT CDVersion (#PCDATA) >
<!ELEMENT CDStatus (#PCDATA) >
<!ELEMENT CDURL (#PCDATA) >
<!ELEMENT CDUses (CDName*) >
<!ELEMENT CDComment (#PCDATA) >
<!ELEMENT Name (#PCDATA) >
<!ELEMENT CMP (#PCDATA) >
<!-- include dtd for OM objects -->
<!ENTITY % omobjectdtd SYSTEM "omobj.dtd" >
%omobjectdtd;
<!ELEMENT FMP (OMOBJ?) >
<!ELEMENT Example (#PCDATA | OMOBJ)* >
<!ELEMENT CDDefinition (Name,
    (Description | CDComment | CMP | FMP | Example )*) >
<!ELEMENT CD ( CDName | Description | CDReviewDate | CDDate |
    CDVersion | CDStatus | CDURL | CDUses |
    CDComment | Example | CDDefinition )* >
<!-- end of DTD for OM CD -->
```

Figure 5.1: DTD Specification of Content Dictionaries

Description The text occurring in the Description element is used to give a description of the enclosing element, which could be a symbol or the entire Content Dictionary. The content of this element can be any XML text.
CDReviewDate The text occurring in the CDReviewDate element corresponds to the earliest possible revision date of the Content Dictionary. The date formats should be ISO-compliant in the form YYYY-MM-DD, e.g. 1953-09-26.
CDDate The text occurring in the CDDate element corresponds to the date of this version of the Content Dictionary. The date formats should be ISO-compliant in the form YYYY-MMDD, e.g. 1953-09-26.

1999/06/23
new paragraph
1999/11/24
Now just an integre

1999/11/24
New field, formally
'. $y$ ' of version number

1999/06/23
new wording

1999/10/01
Due to lack of inspiration, I added only these few lines

Example The text occurring in the Example element is used to give examples of the enclosing symbol, and can be any XML text. In addition to text the element may contain examples as XmL encoded OpenMath, inside OMOBJ elements. Note that Examples must be with respect to some symbol and cannot be "loose" in the Content Dictionary.
Name The text occurring in the Name element corresponds to the name of the symbol, and is specified as in Chapter 4.
CMP The text occurring in the CMP element corresponds to a property of the symbol. An application which says it understands a Content Dictionary symbol need not understand a commented property of the symbol.
FMP The content of the FMP element also corresponds to a property ${ }^{1}$ of the symbol, however the content of this element must be a valid OpenMath object in the XML encoding. An application which says it understands a Content Dictionary symbol need not understand a formal property of the symbol.

### 5.4 Additional Information

Content Dictionaries contain just one part of the information that can be associated to a symbol in order to stepwise define its meaning and its functionality. OpenMath Signature files, CDGroups, and possibly files of extra mathematical properties, are used to convey the different aspects that as a whole make up a mathematical definition.

### 5.4.1 Signature Files

OpenMath may be used with any type system. One just needs to produce a Content Dictionary which gives the constructors of the type system, and then one may build OpenMath objects representing types in the given type system. These are typically associated with OpenMath objects via the OpenMath attribution constructor.

A Small Type System, called STS, has been designed to give semi-formal signatures to OpenMath symbols and is documented in [10]. The signature file given in Appendix A. 3 is based on this formalism. Using the same mechanism, [5] shows how pure type systems can also be employed to assign types to OpenMath symbols.

### 5.4.1.1 The DTD Specification of Signature Files

Signature Files are XmL documents, hence a valid Signature File should

- be valid according to the DTD given in Figure 5.2,
- adhere to the extra conditions on the content of the elements given in Section 5.4.1.2.

Signature files have a header which specifies the Content Dictionary and determines the type system being used, and the Content Dictionary which contains the symbols for which the signatures are being given. Each signature takes the form of an XML encoded OpenMath object.

[^2]```
<!-- omcds.dtd -->
<!-- ********************************************** -->
<!-- -->
<!-- DTD for OpenMath CD Signatures -->
<!-- (c) EP24969 the ESPRIT OpenMath Consortium -->
<!-- David Carlisle 1999-04-13 -->
<!-- David Carlisle 1999-05-21 -->
<!-- Olga Caprotti 1999-08-25 removed CDComment -->
<!-- -->
<!-- ********************************************* -->
<!-- include dtd for OM objects -->
<!ENTITY % omobjectdtd SYSTEM "omobj.dtd" >
%omobjectdtd;
<!ELEMENT CDSComment (#PCDATA) >
<!ELEMENT CDSReviewDate (#PCDATA) >
<!ELEMENT CDSStatus (#PCDATA) >
<!ELEMENT CDSignatures (CDSComment | CDSReviewDate |
    CDSStatus | Signature )* >
<!ATTLIST CDSignatures cd CDATA #REQUIRED
    type CDATA #REQUIRED >
<!ELEMENT Signature (OMOBJ) >
<!ATTLIST Signature name CDATA #REQUIRED >
<!-- end of DTD for OM CD Signatures -->
```

Figure 5.2: DTD Specification of Signature Files

### 5.4.1.2 Further Requirements of a Signature File

The notion of being a valid Signature File is stronger than merely being successfully parsed by the DTD in Figure 5.2. In this section we define exactly the format of the elements used in Signature Files. Several of the requirements are the same as those on elements of Contents Dictionaries.

CDSignatures The outermost element of the Signature File is characterized by two required attributes that identify the type system and the Content Dictionary whose signatures are defined. The value of the XML attribute type is the name of the Content Dictionary or of the CDGroup (cfg. Section 5.4.2) that represents the type system. The value of the XML attribute cd is the name of the Content Dictionary whose symbols are assigned signatures in this Signature File. Both values are of the form specified in Chapter 4.
CDSComment See CDComment in Section 5.3.2.
CDSreviewDate The text occurring in the CDSReviewDate element corresponds to the earliest possible revision date of the Signature File. The date formats should be ISO-compliant in the form YYYY-MM-DD, e.g. 2000-02-29.
CDSStatus The text occurring in the CDSStatus element corresponds to the status of the Signature File, and can be either official (approved by the OpenMath Society according to the procedure outlined in Section 5.5), experimental (currently being tested), private (used by a private group of OpenMath users) or obsolete (an obsolete Signature File kept only for archival purposes).
Signature The content of the Signature element has to be a valid OpenMath object in XmL encoding as specified in Chapter 4. Additionally, the object must represent a valid type in the type system identified by the XML attribute type of the CDSignature element. See Section 5.4.1.3 for examples.

### 5.4.1.3 Examples

An example of a signature file for the type system STS and the arith1 Content Dictionary is given in Appendix A.3 . Each signature entry is similar to the following one for the OpenMath symbol <OMS cd="arith1" name="plus"/>:

```
<Signature name="plus">
<OMOBJ>
    <OMA>
        <OMS name="mapsto" cd="sts"/>
        <OMA>
        <OMS name="nassoc" cd="sts"/>
        <OMV name="AbelianSemiGroup"/>
        </OMA>
        <OMV name="AbelianSemiGroup"/>
    </OMA>
</OMOBJ>
</Signature>
```


### 5.4.2 CDGroups

The CD Group mechanism is a convenience mechanism for identifying collections of CDs. A CD Group file is an xML document used in the (static or dynamic) negotiation phase where communicating applications declare and agree on the Content Dictionaries which they process. It is a complement, or an alternative, to the individual declaration of Content Dictionaries understood by an application. Note that CD Groups do not affect the OpenMath objects themselves. Symbols in an object always refer to content dictionaries, not groups.

For an application to declare that it "understands CDGroup G" is exactly equivalent to, and interchangable with, the declaration that it "understands Content Dictionaries $x_{1}, x_{2}, \ldots x_{n}$ ", where $x_{1}, \ldots x_{n}$ are the members of CDGroup G.

### 5.4.2.1 The DTD Specification of CDGroups

CDGroups are XML documents, hence a valid CDGroup should

- be valid according to the DTD given in Figure 5.3,
- adhere to the extra conditions on the content of the elements given in Section 5.4.2.2.

Apart from some header information such as CDGroupName and CDGroup version, a CDGroup is simply an unordered list of CDs, identified by name and optionally version number and URL.

### 5.4.2.2 Further Requirements of a CDGroup

The notion of being a valid CDGroup implies that the following requirements on the content of the elements described by the DTD in Figure 5.2 are also met.

CDGroup The XML element CDGroup is the outermost element in a CDGroup document.
CDGroupName The text occurring in the CDGroupName element corresponds to the name of the CDGroup. For the syntactical requirements, see CDName in Section 5.3.2.
CDGroupURL The text occurring in the CDGroupURL element identifies the location of the CDGroup file, not necessarily of the member Content Dictionaries. For the syntactical requirements, see CDURL in Section 5.3.2.
CDGroupDescription The text occurring in the CDGroupDescription element describes the mathematical area of the CDGroup.
CDGroupMember The XML element CDGroupMember encloses the data identifying each member of the CDGroup.
CDName The text occurring in the CDName element corresponds to the name of a Content Dictionary in the CDGroup. For the syntactical requirements, see CDName in Section 5.3.2.
CDVersion The text occurring in the CDVersion element identifies which version of the Content Dictionary isto be taken as member of the CDGroup. This element is optional. In case it is missing, the latest version is the one included in the CDGroup. For the syntactical requirements, see CDVersion in Section 5.3.2.
CDURL The text occurring in the CDURL element identifies the location of the Content Dictionary to be taken as member of the CDGroup. This element is optional. In case it is missing, the location of the CDGroup identified by the element CDGroupURL is assumed. For the syntactical requirements, see CDURL in Section 5.3.2.

1999/06/2
All new, partly
taken from SB paper
1999/10/0
Rephrase sligh

1999/08/26
Added PCDATA for CDGroup

1999/08/01
For consistency, CDGName would be better

1999/08/01
Or the official CD repository?

```
<!-- CDgroup.dtd -->
<!-- ********************************************** -->
<!--
    -->
<!-- DTD for OpenMath CD group -->
<!-- (c) EP24969 the ESPRIT OpenMath Consortium -->
<!-- date = 18.Feb.1999 -->
<!-- author = s.buswell sb@stilo.demon.co.uk -->
<!-- -->
<!-- -->
<!-- available at -->
<!-- http://www.openmath.org/cd/dtd/CDgroup.dtd -->
<!-- -->
<!-- ********************************************** -->
<!-- info on the CD group itself -->
<!ELEMENT CDGroupName (#PCDATA) >
<!ELEMENT CDGroupDescription (#PCDATA) >
<!ELEMENT CDGroupVersion (#PCDATA) >
<!ELEMENT CDGroupURL (#PCDATA) >
<!-- info on the CDs in the group -->
<!ELEMENT CDGroupMember (CDName, CDVersion?, CDURL?) >
<!ELEMENT CDName (#PCDATA) >
<!ELEMENT CDVersion (#PCDATA) >
<!ELEMENT CDURL (#PCDATA) >
<!ELEMENT CDComment (#PCDATA) >
<!-- structure of the group -->
<!ELEMENT CDGroup (CDGroupName, CDGroupDescription,
    CDGroupVersion, CDGroupURL,
    (CDGroupMember | CDComment )* ) >
<!-- end of DTD for OM CDGroup -->
```

Figure 5.3: DTD Specification of CDGroups

1999/10/04
Delete subsec: Note on Symbols, CDs and CDGroups
2000/04/10 Delete examples (MathML
CDGroup is in appendix, core CDGroup no longer exists
1999/08/25
This section to be added
1999/10/04
Delete subsec:
DefMP Files and
XSL
1999/10/04
Rephrase slightly

### 5.5 Content Dictionaries Reviewing Process

The OpenMath Society is responsible for implementing a review and referee process to assess the accuracy of the mathematical content of Content Dictionaries. The status (see CDStatus) and/or the version number (see CDVersion) of a Content Dictionary may change as a result of this review process.

## Chapter 6

## OpenMath Compliance

Applications that meet the requirements specified in this chapter may label themselves as OpenMath compliant. OpenMath compliancy is defined so as to maximize the potential for interoperability amongst OpenMath applications.

### 6.1 Encoding

This standard defines two reference encodings for OpenMath, the binary encoding and XML encoding, defined in chapter 4.
As a minimum, an OpenMath compliant application, which accepts or generates OpenMath objects, must be capable of doing so using the XML encoding. The ability to use other encodings is optional.

### 6.2 Content Dictionaries

An OpenMath compliant application must be able to support the error Content Dictionary defined in Appendix A. 5.
A compliant application must declare the names and version numbers of the Content Dictionaries that it supports. Equivalently it may declare the Content Dictionary Group (or groups) and major version number (not revision number), rather than listing individual Content Dictionaries. Application that support all Content Dictionaries (e.g. renderers) should refer to the implicit CD Group all
If a compliant application supports a Content Dictionary then it must explicitly declare any symbols in the Content Dictionaries that are not supported. Phrasebooks are encouraged to support every symbol in the Content Dictionaries.

Symbols which are not listed as unsupported are supported by the application. The meaning of supported will depend on the application domain. For example an OpenMath renderer should provide a default display for any OpenMath object that only references supported symbols, whereas a Computer Algebra System will be expected to map such an object to a suitable
internal representation, in this system, of this mathematical object. It is expected that the application's phrasebooks for supported Content Dictionaries will be constructed such that propertes of the symbol expressed in the Content Dictionary are respected as far as possible for the given application domain. However OpenMath compliance does not imply any guarantee by the OpenMath Society on the accuracy of these representations.
Content Dictionaries available from the official OpenMath repository at www.openmath.org need only be referenced by name, other Content Dictionaries should be referenced by the URL declared in the CDURL field of the Dictionary. This URL may be used to retrieve the Content Dictionary.
When receiving an OpenMath symbol, e.g. $s$, that is not supported from a supported Content Dictionary, a compliant application will act as if it had received the OpenMath object

```
error(Unhandled_Symbol, s)
```

where Unhandled_Symbol is the symbol from the error Content Dictionary.
Similarly if it receives a symbol, e.g. $s$, from an unsupported Content Dictionary, it will act as if it had received the OpenMath object

```
error(Unsupported_CD, s)
```

Finally if the compliant application receives a symbol from a supported Content Dictionary but with an unknown name, then this must either be an incorrect object, or possibly the object has been built using a later version of the Content Dictionary. In either case, the application will act as if it had received the OpenMath object

```
error(Unexpected_Symbol, s)
```


### 6.3 Lexical Errors

The previous section defines the behaviour of a compliant application upon receiving well formed OpenMath objects containing unexpected symbols. This standard does not specify any behaviour for an application upon receiving ill-formed objects.

## Chapter 7

## Conclusion

The goal of this document is to define the OpenMath standard. The things are addressed by the OpenMath standard are:

- Informal and formal definition of the OpenMath objects.
- Informal and formal definition of the notion of Content Dictionaries.

To do this, OpenMath objects are precisely defined and two encodings are described to represent these objects using xml and binary code. Furthermore, the Document Type Definition for validating Content Dictionaries and OpenMath objects is given.

## Appendix A

## A. 1 The meta Content Dictionary

```
<CD>
<CDName> meta </CDName>
<CDReviewDate> 1999-09-01 </CDReviewDate>
<CDDate> 1999-05-11 </CDDate>
<CDVersion> 1.1a </CDVersion>
<CDStatus> experimental </CDStatus>
<CDURL> http://openmath.nag.co.uk/Projects/openmath/corecd/cd/meta.ocd </CDURL>
<Description>
This is a content dictionary to represent content dictionaries, so
that they may be passed between OpenMath compliant application in a
similar way to mathematical objects.
The information written here is taken from chapter 4 of the current
draft of the "OpenMath Standard".
</Description>
<CDComment>
First Draft 1998 N. Howgrave-Graham.
Modified 1999-02-13 R Timoney to fix errors and omissions.
Modified 1999-03-28 D Carlisle to change description of Signature.
Rewritten 1999-05-07 D Carlisle.
Modified 1999-05-11 D Carlisle. Added CDDate and CDVersion.
</CDComment>
```

```
<CDDefinition>
<Name> CD </Name>
<Description>
The top level element for the Content Dictionary. It just acts
as a container for the elements described below.
</Description>
</CDDefinition>
```

```
<CDComment>
For those that do not have access to the DTD, the elements
allowed in a Content Dictionary are the following
(in no particular order):
<![CDATA[
<CD>
<CDName> </CDName>
<Description> </Description>
<CDReviewDate> </CDReviewDate>
<CDDate> </CDDate>
<CDVersion> </CDVersion>
<CDStatus> </CDStatus>
<CDURL>? </CDURL>
<CDUses>? <CDUses>
<CDDefinition>*
<Name> </Name>
<Description>* </Description>
<Signature>? </Signature>
<Example>* </Example>
<FMP>* </FMP>
<CMP>* </CMP>
<Presentation>? </Presentation>
</CDDefinition>
]]>
where an asterisk (?) denotes it can repeated 0 or 1 times, and a star
(*) denotes 0 or more times.
</CDComment>
<CDDefinition>
<Name> CDName </Name>
<Description>
An element which contains the string corresponding to the name of the CD.
Here and elsewhere white space occurring at the begining or end of the
string will be ignored. The string must match the syntax for
CD names given in the OpenMath Standard.
</Description>
</CDDefinition>
<CDDefinition>
<Name> CDURL </Name>
<Description>
An optional element.
If it is used it contains a string representing the URI where the
cannonical reference copy of this CD is stored.
</Description>
</CDDefinition>
<CDDefinition>
<Name> Example </Name>
<Description>
An element which contains an arbitrary number of children,
```

```
each of which is either a string or an XML encoding of an OpenMath Object.
These children give examples in natural language, or in OpenMath, of the
enclosing symbol definition.
</Description>
</CDDefinition>
<CDDefinition>
<Name> CDDate </Name>
<Description>
An element which contains a date as a string in the ISO-8601
YYYY-MM-DD format. This gives the date at which the Content Dictionary
was last edited.
</Description>
</CDDefinition>
<CDDefinition>
<Name> CDVersion </Name>
<Description>
An element which contains a version string for the CD.
This should be of the form 1.2a with the letter just being changed
for "cosmetic" edits to the file, and the major or minor version numbers
being changed for structural changes that affect the OpenMath Objects
that may use this CD.
</Description>
</CDDefinition>
<CDDefinition>
<Name> CDReviewDate </Name>
<Description>
An element which contains a date as a string in the ISO-8601
YYYY-MM-DD format. This gives the date at which the Content Dictionary
is next scheduled for review. It should be expected to be stable
until at least this date.
</Description>
</CDDefinition>
<CDDefinition>
<Name> CDStatus </Name>
<Description>
An element giving information on the status of the CD.
The content of the element must be one of the following.
official (approved by the OpenMath Society),
experimental (currently being tested),
private (used by a private group of OpenMath users), or
obsolete (an obsolete CD kept only for archival purposes).
</Description>
</CDDefinition>
```

```
<CDDefinition>
<Name> CDUses </Name>
<Description>
An element which contains zero or more CDNames which correspond
to the CDs that this CD depends on. This makes an inheritance
structure for CDs. If the CD is dependent on any other CDs they must
be present here.
</Description>
</CDDefinition>
<CDDefinition>
<Name> Description </Name>
<Description>
An element which contains a string corresponding to the
description of either the CD or the symbol
(depending on which is the enclosing element).
</Description>
</CDDefinition>
<CDDefinition>
<Name> Name </Name>
<Description>
An element containing the string corresponding to the name of
the symbol being defined. This must match the syntax for
symbol names given in the OpenMath Standard.
</Description>
</CDDefinition>
<CDDefinition>
<Name> Signature </Name>
<Description>
An optional element which contains the XML encoding
of an OpenMath object corresponding to
the type of the symbol being defined.
This is not used in the current CD as the signatures are specified
separately in signature files, to allow different type systems to
be used.
</Description>
</CDDefinition>
<CDDefinition>
<Name> Presentation </Name>
<Description>
An optional element (which may be repeated many times) which contains
a string corresponding to a way of presenting the symbol being defined.
</Description>
</CDDefinition>
<CDDefinition>
<Name> CMP </Name>
<Description>
An optional element (which may be repeated many times) which contains
a string corresponding to a property of the symbol being
```

```
defined.
</Description>
</CDDefinition>
<CDDefinition>
<Name> FMP </Name>
<Description>
An optional element which contains an arbitrary number of children,
each of which is either a string or an XML encoding of an OpenMath Object.
Each child corresponds to to a property of the symbol being
defined.
</Description>
</CDDefinition>
</CD>
```


## A. 2 The arith1 Content Dictionary File

```
<CD>
<CDName> arith1 </CDName>
<CDURL> http://openmath.nag.co.uk/Projects/openmath/corecd/cd/arith1.ocd </CDURL>
<CDReviewDate> 1999-09-01 </CDReviewDate>
<CDStatus> experimental </CDStatus>
<CDDate> 1999-07-15 </CDDate>
<CDVersion> 1.02 </CDVersion>
<CDUses>
    <CDName>alg1</CDName>
    <CDName>fns1</CDName>
    <CDName>integer</CDName>
    <CDName>interval</CDName>
    <CDName>logic1</CDName>
    <CDName>quant1</CDName>
    <CDName>relation1</CDName>
</CDUses>
<Description>
This CD defines symbols for common arithmetic functions.
</Description>
<CDDefinition>
<Name> plus </Name>
<Description>
An nary commutative function plus.
</Description>
<CMP> a + b = b + a </CMP>
<FMP>
<OMOBJ>
    <OMBIND>
        <OMS cd="quant1" name="forall"/>
        <OMBVAR>
            <OMV name="a"/>
            <OMV name="b"/>
        </OMBVAR>
        <OMA>
            <OMS cd="relation1" name="eq"/>
            <OMA>
                <OMS cd="arith1" name="plus"/>
                <OMV name="a"/>
                <OMV name="b"/>
            </OMA>
            <OMA>
                    <OMS cd="arith1" name="plus"/>
                    <OMV name="b"/>
                    <OMV name="a"/>
            </OMA>
        </OMA>
    </OMBIND>
</OMOBJ>
</FMP>
</CDDefinition>
```

```
<CDDefinition>
<Name> unary_minus </Name>
<Description>
This symbol denoting unary minus. Ie
the additive inverse.
</Description>
<CMP> a + (-a) = 0 </CMP>
<FMP>
<OMOBJ>
    <OMBIND>
        <OMS cd="quant1" name="forall"/>
        <OMBVAR>
            <OMV name="a"/>
        </OMBVAR>
        <OMA>
            <OMS cd="relation1" name="eq"/>
            <OMA>
                <0MS cd="arith1" name="plus"/>
                <OMV name="a"/>
                <OMA>
                    <OMS cd="arith1" name="unary_minus"/>
                    <OMV name="a"/>
                    </OMA>
                </OMA>
                <OMS cd="alg1" name="zero"/>
            </OMA>
    </OMBIND>
</OMOBJ>
</FMP>
</CDDefinition>
<CDDefinition>
<Name> minus </Name>
<Description>
The binary minus symbol. This is equivalent to adding the
additive inverse.
</Description>
<CMP> a - b = a + (-b) </CMP>
<FMP>
<OMOBJ>
    <OMBIND>
        <OMS cd="quant1" name="forall"/>
        <OMBVAR>
            <OMV name="a"/>
            <OMV name="b"/>
        </OMBVAR>
        <OMA>
            <OMS cd="relation1" name="eq"/>
            <OMA>
                    <OMS cd="arith1" name="minus"/>
                    <OMV name="a"/>
                    <OMV name="b"/>
            </OMA>
            <OMA>
```

```
            <OMS cd="arith1" name="plus"/>
            <OMV name="a"/>
            <OMA>
                    <OMS cd="arith1" name="unary_minus"/>
                    <OMV name="b"/>
            </OMA>
            </OMA>
        </OMA>
    </OMBIND>
</OMOBJ>
</FMP>
</CDDefinition>
<CDDefinition>
<Name> times </Name>
<Description>
This is an n-ary multiplication function.
</Description>
</CDDefinition>
<CDDefinition>
<Name> divide </Name>
<Description>
This is the (binary) division function that denotes the first argument
right-divided by the second, i.e. divide(a,b)=a*inverse(b). It is the
inverse of multiplication function as commented below.
</Description>
<CMP> whenever not(a=0) then a/a = 1 </CMP>
<FMP>
<OMOBJ>
    <OMBIND>
        <OMS cd="quant1" name="forall"/>
        <OMBVAR>
            <OMV name="a"/>
        </OMBVAR>
        <OMA>
            <OMS cd="logic1" name="implies"/>
            <0MA>
                <OMS cd="relation1" name="neq"/>
                <OMV name="a"/>
                <OMS cd="alg1" name="zero"/>
            </OMA>
                <OMA>
                    <OMS cd="relation1" name="eq"/>
                    <OMA>
                    <OMS cd="arith1" name="divide"/>
                    <OMV name="a"/>
                    <OMV name="a"/>
                    </OMA>
                    <OMS cd="alg1" name="one"/>
                </OMA>
        </OMA>
    </OMBIND>
</OMOBJ>
```

```
</FMP>
</CDDefinition>
<CDDefinition>
<Name> power </Name>
<Description>
A binary powering function. The first argument is raised to the power
of the second argument. When the second argument is not an integer
care should be taken to the meaning of this function; however it is
here to represent general powering.
</Description>
</CDDefinition>
<CDDefinition>
<Name> conjugate </Name>
<Description>
A unary function to give the complex conjugate of its argument
</Description>
</CDDefinition>
<CDDefinition>
<Name> abs </Name>
<Description>
A unary function to give the absolute value of its argument. This is
used for the absolute size of complex numbers as well (commonly
referred to as mod).
</Description>
</CDDefinition>
<CDDefinition>
<Name> root </Name>
<Description>
A binary function to give roots. The first argument is "lowered" to
the root of the second argument. This can be viewed as the inverse of
powering as commented below.
Care should be taken to the meaning of this function (i.e. which root
is being taken); however it is here to represent the general notion of
taking n'th roots.
</Description>
<CMP> power(root(a,n),n) = a </CMP>
<FMP>
    <OMOBJ>
        <OMBIND>
            <OMS cd="quant1" name="forall"/>
            <OMBVAR>
                    <OMV name="a"/>
                    <OMV name="n"/>
            </OMBVAR>
            <OMA>
                <OMS cd="relation1" name="eq"/>
                <OMA>
                    <OMS cd="arith1" name="power"/>
                    <OMA>
```

```
                    <OMS cd="arith1" name="root"/>
                    <OMV name="a"/>
                    <OMV name="n"/>
                    </OMA>
                <OMV name="n"/>
            </OMA>
            <OMV name="a"/>
        </OMA>
        </OMBIND>
    </OMOBJ>
</FMP>
</CDDefinition>
```


## <CDDefinition>

<Name> sum </Name>
<Description>
Form taking two arguments, first being an integer interval giving the
range of summation, second being the function to be summed. Compare
defint in calculus CD.
</Description>
<Example>
<OMOBJ>
<OMA>
<OMS cd="arith1" name="sum"/>
<OMA>
<OMS cd="interval" name="integer_interval"/>
<OMI> 1 </OMI>
<OMI> 10 </OMI>
</OMA>
<OMBIND>
<OMS cd="fns1" name="lambda"/>
<OMBVAR>
<OMV name="x"/>
</OMBVAR>
<OMA>
<OMS cd="arith1" name="divide"/>
<OMI>1</OMI>
<OMV name="x"/>
</OMA>
</OMBIND>
</OMA>
</OMOBJ>
</Example>
</CDDefinition>
<CDDefinition>
<Name> product </Name>
<Description>
Form taking two arguments, first being an integer interval giving the range of summation, second being the function to be multiped.

```
Compare defint in calculus CD.
</Description>
<Example>
<OMOBJ>
    <OMA>
        <OMS cd="relation1" name="eq"/>
        <OMA>
            <OMS cd="integer" name="factorial"/>
            <OMV name="n" />
        </OMA>
        <OMA>
            <OMS cd="arith1" name="product"/>
                    <OMA>
                        <OMS cd="interval" name="integer_interval"/>
                        <OMI> 1 </OMI>
                    <OMV name="n"/>
                    </OMA>
            <OMBIND>
                    <OMS cd="fns1" name="lambda"/>
                        <OMBVAR>
                            <OMV name="i"/>
                    </OMBVAR>
                    <OMV name="i"/>
            </OMBIND>
        </OMA>
    </OMA>
</OMOBJ>
</Example>
</CDDefinition>
</CD>
```


## A. 3 The arith1 STS Signature File

1999/07/16

```
<CDSignatures type="sts" cd="arith1">
```

<CDComment>
Date: 1999-04-13
Author: David Carlisle
</CDComment>
<Signature name="plus">
<OMOBJ>
    <OMA>
    <OMS name="mapsto" cd="sts"/>
    <OMA>
        <OMS name="nassoc" cd="sts"/>
        <OMV name="AbelianSemiGroup"/>
    </OMA>
    <OMV name="AbelianSemiGroup"/>
    </OMA>
</OMOBJ>
</Signature>
<Signature name="unary_minus">
<OMOBJ>
    <OMA>
    <OMS name="mapsto" cd="sts"/>
    <OMV name="AbelianGroup"/>
    <OMV name="AbelianGroup"/>
    </OMA>
</OMOBJ>
</Signature>
<Signature name="minus">
<OMOBJ>
    <OMA>
    <OMS name="mapsto" cd="sts"/>
    <OMV name="AbelianGroup"/>
    <OMV name="AbelianGroup"/>
    <OMV name="AbelianGroup"/>
    </OMA>
</OMOBJ>
</Signature>
<Signature name="times">
<OMOBJ>
    <OMA>
        <OMS name="mapsto" cd="sts"/>
    <OMA>
        <OMS name="nassoc" cd="sts"/>
        <OMV name="AbelianSemiGroup"/>
    </OMA>
    <OMV name="AbelianSemiGroup"/>
    </OMA>
</OMOBJ>
</Signature>
<Signature name="divide"> <OMOBJ>
<OMA>
<OMS name="mapsto" cd="sts"/>
<OMV name="AbelianGroup"/>
<OMV name="AbelianGroup"/>
<OMV name="AbelianGroup"/>
</OMA>
</OMOBJ>
</Signature>
<Signature name="power">
<OMOBJ>
<OMA>
<OMS name="mapsto" cd="sts"/>
<OMS name="NumericalValue" cd="sts"/>
<OMS name="NumericalValue" cd="sts"/>
<OMS name="NumericalValue" cd="sts"/>
</OMA>
</OMOBJ>
</Signature>
<Signature name="conjugate">
<OMOBJ>
<OMA>
<OMS name="mapsto" cd="sts"/>
<OMS name="C" cd="setname"/>
<OMS name="C" cd="setname"/>
</OMA>
</OMOBJ>
</Signature>
<Signature name="abs">
<OMOBJ>
<OMA>
<OMS name="mapsto" cd="sts"/>
<OMS name="C" cd="setname"/>
<OMV name="R" cd="setname"/>
</OMA>
</OMOBJ>
</Signature>
<Signature name="root">
<OMOBJ>
<OMA>
<OMS name="mapsto" cd="sts"/>
<OMS name="NumericalValue" cd="sts"/>
<OMS name="NumericalValue" cd="sts"/>
<OMS name="NumericalValue" cd="sts"/>
</OMA>
</OMOBJ>
</Signature>

```
<Signature name="sum" >
<OMOBJ>
    <OMA>
    <OMS name="mapsto" cd="sts" />
    <OMV name="IntegerRange" />
    <OMA>
    <OMS name="mapsto" cd="sts" />
    <OMS name="Z" cd="setname" />
    <OMV name="AbelianMonoid" />
    </OMA>
    <OMV name="AbelianMonoid" />
</OMA>
</OMOBJ>
</Signature>
<Signature name="product" >
<OMOBJ>
    <OMA>
    <OMS name="mapsto" cd="sts" />
    <OMV name="IntegerRange" />
    <OMA>
        <OMS name="mapsto" cd="sts" />
        <OMS name="Z" cd="setname" />
        <OMV name="AbelianMonoid" />
        </OMA>
        <OMV name="AbelianMonoid" />
    </OMA>
</OMOBJ>
</Signature>
</CDSignatures>
```


## A. 4 The MathML CDGroup

1999/08/26
ADD MathML
CDGroup

```
<CDGroup>
<CDGroupName>mathml</CDGroupName>
<CDGroupVersion>1.0</CDGroupVersion>
<CDGroupURL>
http://www.nag.co.uk/Projects/openmath/corecd/cdgroups/mathml.ocd</CDGroupURL>
<CDGroupDescription>MathML Compatibility CD Group</CDGroupDescription>
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It was created by S.Buswell on 29 March 1999.</CDComment>
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```
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<CDGroupMember>
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(which can be seen as nullary constructors)</CDComment>
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<CDGroupMember>
<CDName>stats1</CDName>
<CDURL>http://www.nag.co.uk/Projects/openmath/corecd/cd/stats1.ocd</CDURL></CDGroupMember>
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<CDComment>Types that are needed in openMath for MathML alignment</CDComment>
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Semantic mapping constructs.</CDComment>
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## A. 5 The error Content Dictionary

## <CD>

<CDName> error </CDName>
<CDURL> http://www.nag.co.uk/Projects/openmath/corecd/cd/error.ocd </CDURL>
<CDReviewDate> 2000-09-01 </CDReviewDate>

```
<CDStatus> experimental </CDStatus>
<CDDate> 2000-04-18 </CDDate>
<CDVersion> 1 </CDVersion>
<CDRevision> 0 </CDRevision>
<CDUses>
<CDName> setname2 </CDName>
<CDName> arith1 </CDName>
<CDName> specfun1 </CDName>
</CDUses>
<CDDefinition>
<Name> unhandled_symbol </Name>
<Description>
This symbol represents the error which is raised when an application
reads a symbol which is present in the mentioned content
dictionary, but which it has not implemented.
When receiving such a symbol, the application should act as if it had
received the OpenMath error object constructed from unhandled_symbol
and the unhandled symbol as in the example below.
</Description>
<Example>
The application does not implement the quaternions:
<OMOBJ>
    <OME>
        <OMS cd="error" name="unhandled_symbol"/>
        <OMS cd="setname2" name="H"/>
    </OME>
</OMOBJ>
</Example>
</CDDefinition>
<CDDefinition>
<Name> unexpected_symbol </Name>
<Description>
This symbol represents the error which is raised when an application
reads a symbol which is not present in the mentioned content dictionary.
When receiving such a symbol, the application should act as if it had
received the OpenMath error object constructed from unexpected_symbol
and the unexpected symbol as in the example below.
</Description>
<Example>
The application received a mistyped symbol
<OMOBJ>
    <OME>
        <OMS cd="error" name="unexpected_symbol"/>
        <OMS cd="arith1" name="plurse"/>
    </OME>
</OMOBJ>
</Example>
</CDDefinition>
```

<CDDefinition>
<Name> unsupported_CD </Name>
<Description>
This symbol represents the error which is raised when an application reads a symbol where the mentioned content dictionary is not present.

When receiving such a symbol, the application should act as if it had received the OpenMath error object constructed from unsupported_CD and the symbol from the unsupported Content Dictionary as in the example below.
</Description>
<Example>
The application does not know about the CD specfun1
<OMOBJ>
<OME>
<OMS cd="error" name="unsupported_CD"/>
<OMS cd="specfun1" name="BesselJ"/>
</OME>
</OMOBJ>
</Example>
</CDDefinition>
</CD>

## Bibliography

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[5] Olga Caprotti and Arjeh M. Cohen. A Type System for OpenMath. OpenMath Deliverable 1.3.2b, OpenMath Esprit Consortium, http://www.nag.co.uk/projects/OpenMath.html, February 1999.
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[12] Iso 7-bit coded character set for information interchange. ISO 646:1983, 1983.
[13] Nico Poppelier, Robert Miner, Patrick Ion, David Carlisle, Ron Ausbrooks, Stephen Buswell, Stéphane Dalmas, Stan Devitt, Angel Diaz, Roger Hunter, Bruce Smith, Neil Soiffer, Robert Sutor, and Stephen Watt. Mathematical Markup Language (MathML) 2.0 Specification. W3C Working Draft 20000328, April 1998. Available at http://www.w3.org/TR/ REC-MathML2/.
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[17] F. Yergeau. UTF-8, a transformation format of ISO 10646. RFC 2279, January 1998. Alis Technologies.

## Appendix B

## Change Log

| 1999/06/20 OC.............................. page 28 now we have this numbering mechanism, should it be documented? | 1999/07/16 DPC................... page 51 |
| :---: | :---: |
|  | ADD arith1 signature file |
|  | 33 |
| 1999/06/20 OC...................... page 33 | sts is not valid wrt DTD |
| All new, partly taken from SB paper | 1999/08/01 OC....................... page 33 |
| 1999/06/20 OC....................... page 34 | This notion might be too strict, it also need |
| Does | CDUses possibly |
| 1999/06/22 OC................................. page 28 new paragraph | 1999/08/01 OC................................... 34 For consistency, CDGName would be better |
| 1999/06/23 OC.................... page 7 | age 34 |
| This |  |
| 1999/06/23 DPC.......................... page 30 new paragraph | 1999/08/24 OC......................................... 4 Changed title |
| $\begin{aligned} & 1999 / 06 / 23 \mathrm{DPC} \ldots \ldots \ldots \ldots \ldots \text {. . . . . . . . . page } 30 \\ & \text { new wording } \end{aligned}$ | $99 / 08 / 24 \text { OC...................................... } 5$ <br> w section |
| $\text { 1999/06/23 OC ............................. . page } 30$ <br> new description | 1999/08/24 OC.............................. page 6 Note on encodings and possibility of other en- |
| 1999/06/24 DPC.................... page 20 |  |
| New attrvar production | 1999/08/24 OC...................... page 9 |
| 1999/07/16 DPC.................... page 4 | Reshuffled the sections on OM Objects |
| Extend History slightly | page 10 |
| 1999/07/16 DPC................... page 4 | Cleaned up Attrib |
| Reword to reflect birth of OM Society | 8/24 OC................... . page 10 |
| 1999/07/16 DPC................... page 5 | Condensed Informal and Notes |
| Final conclusion paragraph removed | 99/08/24 OC..................... page 13 |
| 1999/07/16 DPC....................... page 9 Restructure the definition of OM Objects | Removed reference to syntactic class of an attributed variable |
| page 15 | 1999/08/24 OC...................... page 27 |
| White space allowed in integer strings | More motivation on design of CDs |

[^3]

1999/09/21 DPC page 15
Restrict empty element syntax
1999/09/21 DPC........................ page 20
New section on embedding OM in XML documents

1999/09/22 DPC......................... page 13
Paragraph moved from previous section
1999/09/22 DPC page 13
Remove classification of suggested error types, does not fit current CD scheme

1999/09/22 DPC......................... page 15
White space allowed in integer strings
1999/10/01 OC............................. page 8
Removed mention to DefMP files
1999/10/01 OC
page 30
Due to lack of inspiration, I added only these few lines

1999/10/04 DPC ............................ page 12
Rephrase slightly
1999/10/04 DPC........................... page 26
Rephrase slightly
1999/10/04 DPC ............................ page 28
Rephrase slightly
1999/10/04 DPC ......................... page 31
Rephrase slightly
1999/10/04 DPC ............................ page 33
Rephrase slightly
1999/10/04 DPC .......................... page 36
Delete subsec: DefMP Files and XSL
1999/10/04 DPC
page 36
Delete subsec: Note on Symbols, CDs and CDGroups

1999/10/04 DPC ............................. page 36
Rephrase slightly
1999/10/21 OC........................... page 11
New tree figure, suggested by Andreas Strotmann
1999/11/24 DPC ......................... page 30
New field, formally '. y' of version number
1999/11/24 DPC .......................... page 30
Now just an integre1999/11/24 DPC/OCpage 37
New chapter, after discussions at Esprit Open-Math meeting in Bath
2000/03/20 DPC ..... page 20Namespace URI, as discussed on OM Soc list2000/04/10 DPC
$\qquad$Reword2000/04/10 DPCpage 10Add integer and float2000/04/10 DPCpage 10Change Example2000/04/10 DPCpage 28
MathML 22000/04/10 DPCpage 36
Delete examples (MathML CDGroup is in ap-pendix, core CDGroup no longer exists


[^0]:    Page 4 of 62

[^1]:    Page 6 of 62

[^2]:    ${ }^{1}$ It corresponds to a theorem of a theory in some formal system.

[^3]:    Page 60 of 62

